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Error Discounting in Probabilistic Category Learning

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Abstract

Some current theories of probabilistic categorization assume that people gradually attenuate their learning in response to unavoidable error. However, existing evidence for this error discounting is sparse and open to alternative interpretations. We report two probabilistic-categorization experiments that investigated error discounting by shifting feedback probabilities to new values after different amounts of training. In both experiments, responding gradually became less responsive to errors, and learning was slowed for some time after the feedback shift. Both results are indicative of error discounting. Quantitative modeling of the data revealed that adding a mechanism for error discounting significantly improved the fits of an exemplar-based and a rule-based associative learning model, as well as of a recency-based model of categorization. We conclude that error discounting is an important component of probabilistic learning.

Error Discounting in Probabilistic Category Learning

Throughout daily life, people make decisions based on uncertain cues that predict multiple outcomes: Doctors make diagnoses based on symptoms that are indicative of several potential diseases; dark clouds do not always bring rain, yet an overcast day is more likely to be associated with precipitation than sunshine; and despite injuries, trades, and home-field advantages, we learn to predict which teams are likely to win on the weekend. Accurate prediction requires learning probabilistic relationships between cues and uncertain outcomes (Kruschke & Johansen, 1999). Probabilistic categorization is one form of probability learning in which the to-be-predicted outcomes are discrete categorical values. Unlike its deterministic counterpart, in probabilistic categorization the same stimuli belong to multiple categories albeit with differing probabilities. In consequence, the same response to an identical stimulus can be reinforced as either correct or incorrect on a probabilistic basis (Kruschke & Johansen, 1999). This corrective feedback is often believed to drive category learning (Gluck & Bower, 1988a, 1988b; Rumelhart, Hinton, & Williams, 1986).

In deterministic category learning, people often perform sufficiently well to eliminate error completely. In probabilistic categorization, by contrast, avoiding errors and achieving perfect accuracy is by definition impossible. On that basis, Kruschke and Johansen (1999) proposed that during probabilistic categorization, people may eventually accept a certain level of unavoidable error and progressively slow their learning, a phenomenon known as error discounting. Kruschke and Johansen (1999) instantiated error discounting within a computational model, called RASHNL (Rapid Attention SHifts 'N' Learning). RASHNL includes an error discounting mechanism based on an "annealing" process, which works by reducing learning rates across time (e.g., Amari, 1967; Heskes & Kappen, 1991; Murata, Kawanabe, Ziehe, Müller, & Amari, 2002). This decline in learning rates causes a gradual decrease in the rate at which RASHNL updates associations. Thus, as learning progresses, errors are increasingly discounted because they contribute less and less to learning. The annealed learning provides the model with various advantages over a fixed learning rate: For example, annealing allows for large response shifts early in training, to quickly adapt to the environment, while the slowing of learning later in training permits the fine-tuning of probabilistic responding (e.g., Amari, 1967; Heskes & Kappen, 1991; Murata et al., 2002).

This article critically examines the notion of error discounting and seeks supporting evidence for its operation in probabilistic categorization. We start by examining existing evidence for discounting and find it to be surprisingly sparse. We next present two probabilistic categorization experiments in which the reinforcement probabilities for all stimuli were shifted during training, with the abruptness and timing of the shift varying between experiments and conditions. We present two forms of analysis to confirm the presence of error discounting. First, we examine decisional-recency effects in categorization. The analysis of decisional recency, as developed by Jones, Love, and Maddox (2006), pinpoints how people adjust their behavior in response to feedback from the immediately preceding trial. Using an extension of Jones et al. (2006)'s original analysis, we show that participants gradually come to ignore error feedback when making category decisions, and their sensitivity to feedback is only gradually restored after reinforcement probabilities change. Second, we applied four models of categorization to the results: An exemplar model without an associative-learning mechanism that bases decisions on a similarity comparison to all previously stored exemplars (the Generalized Context Model, GCM; Nosofsky, 1986), a model that utilizes only recency information (the Memory and Context model, MAC; Stewart, Brown, & Chater, 2002), a second exemplar model based on associative learning (RASHNL; Kruschke & Johansen, 1999), and a rule-based model that also included associative learning (derived from the hybrid Attention to Rules and Instances in a Unified Model, ATRIUM; Erickson & Kruschke, 1998, 2002). We find that only the associative-learning models provide an accurate account of the data. Additionally, the MAC, RASHNL, and ATRIUM models provide a significantly better account if they incorporate an error-discounting mechanism. We conclude that error discounting deserves to be given a prominent theoretical role in probabilistic categorization.

Error Discounting in Probabilistic Environments

Examination of the applied literature on performance in complex environments indicates that ignoring unreliable cues is a common risk in industrial settings (Lorenz & Parasuraman, 2007; Moray, 2007) and aviation (Wickens, 2007). However, the existing evidence for error discounting is equivocal. Empirical support for error discounting involves the finding that the delayed introduction of cues reduces their utilization (Edgell, 1983; Edgell & Morrissey, 1987). If cues suddenly become relevant after an initial period of non-relevance, people use them less than equivalent cues that were relevant from the outset. That is, people fail to notice a change in the environment and fail to learn (much) about a highly relevant but new cue (Kruschke & Johansen, 1999). Although this finding is suggestive of error discounting, other explanations, such as those for conventional blocking (Kamin, 1969; Rescorla & Wagner, 1972), cannot be ruled out. Blocking occurs when a previously learned association between a cue, A, and an outcome, X, prevents learning of a new association between a second cue, B, and X, when trained in the presence of A (Kruschke & Blair, 2000). For example, after learning to associate a beep with an electric shock, further training with the conjunction of (the same) beep and a (novel) light (i.e., both the beep and the light now predict the ensuing shock) does not lead to any learning about the light. The light on its own will show little if any evidence of having been learned on a final test. Newly-relevant cues in probabilistic learning may be under-utilized because learning about the new cue may have been blocked by the association between the already-predictive cue(s) and the outcome. It is preferable to seek evidence for error discounting in situations in which there is only one stimulus dimension present: with a single cue, blocking cannot occur. The single-cue approach recognizes that if error discounting is present, people should be insensitive to shifts in reinforcement probabilities. For example, if a stimulus is initially placed in category A with 80% probability, and this suddenly shifts to 20% at some point during learning, error discounting implies that people might continue to predominantly assign that stimulus to category A, at least for some time.

Relevance Shifts in Probabilistic Environments

Research on the effects of shifts in reinforcement probabilities dates back several decades (Estes, 1984; Estes & Straughn, 1954; M. P. Friedman et al., 1964; Yelen & Yelen, 1969). However, upon closer inspection, those data turn out to be either ambivalent or of little relevance to the error-discounting notion. In a series of early studies (e.g., Estes & Straughn, 1954; M. P. Friedman et al., 1964; Yelen & Yelen, 1969) participants learned to predict which of two lights would be illuminated on the next trial. The actual probabilities with which the lights were illuminated differed from chance, and by associating the actual outcome (identity of the illuminated light) with their prediction, people's responses gradually came to track the actual event probabilities. Most relevant here is the fact that in all studies those probabilities changed during the experiment. For example, in the study by Yelen and Yelen (1969), one light might be illuminated on 90% of the first 100 trials (and the other light during the remaining 10%), with those probabilities suddenly reversing for the next 100 trials. In virtually all cases, participants were found to adapt to those changes with remarkable speed. At first glance, this outcome might seem to compromise the notion of error discounting. Upon closer inspection, however, the data arguably have little bearing on probabilistic categorization processes because people never associated distinct outcomes to different stimuli: The only "stimulus" in those early studies was the signal that denoted the beginning of a trial, and consequently any observed learning did not necessarily involve the formation of associations between stimuli and outcomes but simple awareness of base-rates of reinforcement. The data of Estes and Straughn (1954), M. P. Friedman et al. (1964), and Yelen and Yelen (1969) thus confirm that people remain sensitive to base-rates even after more than 1,000 trials with varying probabilities (M. P. Friedman et al., 1964); however, they tell us little about error discounting.

More recently, in a closer analogue to probabilistic categorization, Estes (1984) asked participants to choose between two alternatives on each trial. Each alternative, in turn, was associated with two outcomes (success or failure) that were reinforced with some varying probability. For one of the alternatives, the probability of success remained constant at .5 throughout. For the other alternative, the probability of success increased and decreased throughout training following a sine pattern. The results were intriguing: On the one hand, Estes found that participants were generally aware of the changing probability structure for one of the outcomes and responded accordingly (i.e., following the cyclical sine pattern, preferring that alternative whenever its probability of success exceeded .5 and rejecting it whenever it fell below .5). On the other hand, during a final transfer block involving a uniform probability of success of .5 for both alternatives, people persisted with the cyclical pattern and showed little evidence of adaptation.

On balance, the existing data involving relevance shifts are, at best, ambivalent with respect to error discounting. The study that most closely resembled probabilistic categorization (Estes, 1984), simultaneously found evidence both for people's ability to track changing probabilities and against their ability to adapt after prolonged training. The latter outcome is suggestive of error discounting but remains inconclusive.

Present Experiments

The present experiments used a single, quasi-continuous cue that could take on four values; each value was probabilistically associated with two categories. The use of a single cue precluded the possibility of blocking. The principal manipulation in the experiments involved switching the reinforcement probabilities of the two categories part-way through training. Between experiments, we varied the rate at which the reinforcement probabilities were shifted, which was either sudden (Experiment 1) or gradual (Experiment 2). If the reinforcement probabilities are shifted gradually, the increase in error at any point during the shift is limited. That limited increase, in turn, might be more likely to be discounted. By contrast, a sudden shift in the reinforcement contingencies would lead to a large increase in the error signal, potentially alerting participants to the new environment. Within each experiment, we varied the point at which the shift occurred.

Experiment 1

The first experiment used a single sudden shift in reinforcement. Depending on condition, the shift occurred either early, near the middle, or late in the training sequence.

Method

Participants. Sixty-four members of the University of Western Australia campus community participated in the experiment. Participants were randomly assigned to one of three conditions; *early*, *mid*, or *late*. The number of participants in the three conditions was 21, 21, and 22, for the early, mid, and late conditions respectively. Participants received either course credit or A$10 remuneration.

Stimuli and apparatus. The experiment was controlled by a Windows computer using a MATLAB program created with the aid of the Psychophysics toolbox (Brainard, 1997; Pelli, 1991). The task was a single-cue binary choice task that required participants to classify a set of 4 unique items into one of two categories, A or B. To control for potential idiosyncrasies of the stimuli, two different types of stimuli were used, either squares of varying size, or circles with a radial line at varying angles. Within each condition, participants were randomly assigned to stimulus type. For the squares, the 4 training items were of linearly increasing sizes of 44, 66, 88, and 110 mm edge length. The circles were all approximately 115 mm in diameter. The radial line within the circle was presented at increasing angles of 36, 72, 108, and 144 degrees (measured from the vertical). For some participants the radial line was presented in the left hemisphere of the circle whereas for others it was shown in the right hemisphere.

Stimuli were presented in black on a white background. The category membership of each stimulus was determined on a probabilistic basis, with the probability of an item belonging to category A at the outset of training equaling .2, .4, .6, and .8 in increasing order of size (or angle), respectively. Category B probabilities were $1 - P(A)$. Part-way through the experiment, those probabilities were shifted instantaneously, such that the probability of the items belonging to category A changed to .8, .6, .4, and .2. Category B probabilities also shifted, and remained at $1 - P(A)$. The point at which this shift occurred varied between conditions: The shift occurred at the beginning of training block 2, 11, or 16 for the early, mid, and late conditions, respectively.

Procedure. Each trial commenced with a fixation symbol ("+") in the center of the screen for 500 ms. The "+" was replaced by the stimulus, which was presented in the center of the screen along with the response options (i.e., Category A or Category B) underneath. The stimulus and response options remained visible until a response was made. Responses were made by pressing the "F" or "J" keys. Mappings of the two response keys to categories (A or B) were randomly determined for each participant. After each response, feedback ("CORRECT" or "WRONG") was presented on the screen, below the item, for 1,300 ms.

Items were randomly presented in blocks of 40 trials, with each stimulus presented 10 times per block. Participants were trained for 18 blocks altogether. Self-timed breaks, lasting a minimum of 30 seconds, were presented after every four blocks.

Results and Discussion

In probabilistic categorization, above-chance performance can involve one of two primary behaviors, known as probability-matching and maximizing, respectively (Fantino & Esfandiari, 2002; Shanks, Tunney, & McCarthy, 2002). Probability matching is said to occur if people place items into particular categories with a frequency approximately equivalent to the probability of reinforcement: For example, if stimulus j is assigned to category A on 80% of all trials, then people probability match if they respond A with probability .8. By contrast, maximizing is said to occur if people always place an item into the category to which it is most likely to belong. Thus, for the above item j, maximizing implies that instead of responding with A 80% of the time, people would respond with A on all trials. In most situations, people tend to probability match rather than maximize (Fantino & Esfandiari, 2002; Shanks et al., 2002), although people's performance may lie anywhere along that continuum.

To characterize performance, we computed probability-matching (PM) scores for the participants' mean responses during the last pre-shift block(s); Block 1 for the early condition, Blocks 9 and 10 for the mid condition, and Blocks 14 and 15 for the late condition. PM scores were computed based on the following formula:

$$PM_{ij} = (R_i(A|j) - P(A|j)) * SI_j, \qquad (1)$$

where $P(A|j)$ is the training probability for item j, and $R_i(A|j)$ is the probability of participant i responding with category A in response to item j. SI_j is a sign indicator for item j, such that $SI_j = -1$ if $P(A|j) < .5$ and $SI_j = 1$ if $P(A|j) > .5$ (D. Friedman &

Massaro, 1998; Little & Lewandowsky, 2009b). Each participant's PM score was based on the average score across the four items.

For ease of interpretation, the PM scores were then transformed to a scale ranging from 0 (chance responding) to 1 (full maximizing). For the present probability structure, a score of .4 indicates perfect probability matching. (Scores below 0 indicate reversed responding.) To ensure that only participants who learned the probability structures were included, participants with PM scores of less than 0 were removed from consideration ($N = 3, 4$, and 5 in the early, mid, and late conditions, respectively).

Trials with a response time (RT) of less than 150 ms or an RT more than 4 SDs above the participant's mean RT were removed from the analysis. Additionally, any participant who scored more than 10% of all RTs beyond these cutoffs was excluded from further analysis. Based on these criteria, an additional 3 participants were removed from each of the early, mid, and late conditions.

After the removal of all outliers, the mean PM scores for the remaining participants were 0.44 ($SD = 0.33$), 0.58 ($SD = 0.27$), and 0.50 ($SD = 0.26$), for the early, mid, and late conditions, respectively. These scores were significantly above chance (early: $t(14) = 5.21, p < .001$; mid: $t(13) = 8.12$, $p < .001$; late: $t(13) = 7.37$, $p < .001$).

To check whether there was an impact of stimulus type on responding, PM scores were next calculated for each of the 18 blocks. A three-way between-within subjects ANOVA (Condition × Stimulus type × Block), with the block PM scores as the dependent variable, uncovered no significant main effect of stimulus type, $F(1, 37) = 0.44, p > .1$. For the remainder of the analysis, the data were collapsed across stimulus type.

Mean response probabilities for each of the three conditions for each of the items across the 18 training blocks are presented in Figure 1. The figure makes two points: First, it shows that participants were able to adapt to the change in the probability structure in all three conditions. Second, it shows that people approximately probability-matched to all items, at least in the pre-shift blocks. For example, in the mid condition, the item that was reinforced as belonging to category A 80% of the time on pre-shift trials was placed in that category with probability $\simeq .8$, and so on for all other items.

In order to obtain a more concise summary of people's adaptation to the shift, a single score was computed for each block that summarized responding to all four stimuli. By design, $P(A|j)$ across j was linear within a block (i.e., .2, .4, .6, and .8 or vice versa): Given that participants largely probability-matched, as evidenced by the mean PM scores, the slope of the line through $R_i(A|j)$ across j within each subject-block was likely also linear, a possibility confirmed by inspection of responses. The slope of the response probabilities within each block for each participant was determined by regression, and the observed mean response slopes and the slopes of the objective training probabilities for all blocks are presented in Figure 2. The figure shows each of the mean slopes with error bars indicating 95% confidence intervals. The confidence intervals provide statistical confirmation of a range of effects.

From the slope representation in Figure 2, we can observe participants' initial learning and subsequent adaptation to the shift in reinforcement probabilities. Initial learning was extremely rapid: from virtually the first training block onward, confidence intervals bracketed the objective slopes and were nowhere near chance (i.e., slope 0). This implies that most of the initial learning occurred during the first 40 trials.

Following the shift, participants crossed through chance responding by the second post-shift block in all conditions. However, people adapted more slowly to the change in probabilities than they did during initial learning. Inspection of individual responding suggested that the aggregate pattern was representative of behavior at the individual level. A visual indication of variation among individual responding is given by the 95% confidence intervals in Figure 2.

A two-way between-within subjects ANOVA examined whether the conditions differed with respect to the speed of adaptation. Condition (early vs. mid vs. late) was included as a between-subjects factor, training block was included as a within-subjects factor, with slope as the dependent variable. To render the conditions comparable, they were aligned around the shift point, with the final pre-shift block and the two post-shift blocks comprising the levels of the block factor. There was a significant effect of block, $F(2, 80) = 28.74, p < .001, \eta_p^2 = .42$, but no significant effect of condition, $F(2, 40) = 0.29, p > .1$, and no interaction, $F(4, 80) = .32, p > .1$. Thus, the timing of the shift in the training sequence did not significantly affect the rate of adaptation.

We next sought direct evidence for error discounting by examining trial-by-trial response contingencies. Jones et al. (2006) introduced a trial-by-trial analysis technique to detect "decisional-recency" effects in probabilistic categorization. Specifically, Jones et al. showed that when the stimulus on the current trial was identical to the immediately preceding one, people were biased toward giving the same category response as the feedback presented on the previous trial. As the perceptual difference between the current and the previous stimulus increased, people's response on the current trial was increasingly more likely to *differ* from the preceding category feedback. This decisional recency is consistent with the notion that people's responding is, at least in part, driven by the perceptual contrast between successive stimuli (cf. Stewart et al., 2002). In a nutshell, when a stimulus was repeated, people responded with the category just reinforced whereas they shifted to the alternate category when a stimulus differed dramatically from its immediate predecessor. (Intermediate stimulus differences caused intermediate tendencies to shift.)

Here, we extend Jones et al. (2006)'s technique to explore the existence of error discounting. Whereas Jones et al. focused on the impact of category feedback—irrespective of whether the response was correct or not—on responses to the following stimulus, we examined whether people's response on trial n was affected by whether or not their response on trial $n - 1$ was *correct.* That is, whereas Jones et al. considered only the nature of the feedback on trial $n - 1$, we additionally determined whether or not the probabilistic feedback $n - 1$ was identical to the response $n - 1$.

If people's responding is subject to decisional recency, then the following contingencies should be observed. If the same stimulus is repeated on two successive trials, people should repeat their response if it was correct on the first trial but they should shift to the alternate response following an error. Conversely, if the stimuli on two successive trials are maximally different, then an error should be followed by the *same* response whereas a correct response should be followed by a shift. (Intermediate stimulus differences should again give rise to intermediate response tendencies.) If people gradually come to discount errors, then those response contingencies should decrease across training at least up to the shift in reinforcement probabilities. After the shift, those response contingencies might re-emerge, albeit perhaps slowly.

Figure 3 shows the trial-by-trial contingencies for the mid and late conditions combined across training blocks. (The early condition was omitted because it only comprised a single pre-shift training block.) The top panel shows the probability with which people shifted their response from one trial to the next, P(shift), for trial pairs involving the same stimulus. The bottom panel shows P(shift) for maximally different stimuli. P(shift) was computed by aggregating across trials and participants simultaneously. In order to render the conditions comparable, the data were "end-point aligned," such that the first 5 blocks of the mid condition were aligned with the first 5 blocks of the late condition. Likewise, the final 5 pre-shift blocks in the mid condition, Blocks 6 to 10, were aligned with the last 5 pre-shift blocks in the late condition, viz. Blocks 11 to 15. After alignment, block numbers were zero-centered on the last pre-shift block.

The figure permits several conclusions: First, there is clear evidence for decisional recency because for an identical stimulus (top panel), people tend to shift to the alternate category following an error but they repeat the same category after a correct response. Those response tendencies are reversed for the maximally different stimuli (bottom panel). Second, there is evidence for error discounting because during the pre-shift blocks, P(shift) gradually decreases following an error when the items are the same, and increases for items that are different. That gradual process is reversed once reinforcement probabilities are experimentally shifted and people become more sensitive to errors. We provide statistical confirmation of this pattern in a joint analysis of both experiments once all data have been presented.

Experiment 2

In Experiment 1, we focused on the consequences of sudden shifts in the probabilistic environment. The purpose of Experiment 2 was to explore how people respond to more gradual changes in the probabilistic structure. The consequences of error discounting may be more pronounced and more prolonged when the probabilistic environment shifts more gradually.

Method

Participants. The participants were 38 members of the University of Western Australia community. Participants were randomly assigned to the two conditions, early ($n = 18$) and late ($n = 20$). Of these participants, 9 were presented with square stimuli, and 29 were presented with the circle and radial line stimuli. Participants received either course credit or A$10 remuneration.

Stimuli and apparatus. The stimuli and apparatus were almost identical to that in Experiment 1, with the following exceptions. As in Experiment 1, the feedback was given on a probabilistic basis. Initially, the probability of an item belonging to category A was .2, .4, .6, and .8, respectively, in increasing order of square size or radial line position. Category B probabilities were $1 - P(A)$. In the early condition, the probabilities shifted from their starting values at the beginning of Block 7, viz. to .35, .45, .55, and .65, respectively, for the four items. At the beginning of Block 9, the probabilities shifted through chance to .65, .55, .45, and .35, before finally settling at .8, .6, .4, and .2 from the beginning of Block 11. The probability of an item being in category B was always equal to $1 - P(A)$. In the late condition, the probabilities shifted under the same schedule, except that the shift occurred later in the sequence, beginning with Block 11 and finishing with Block 15. The procedure was identical to Experiment 1.

Results and Discussion

PM scores were calculated over Blocks 5 and 6 for the Early condition, and 9 and 10 for the Late condition. Two and five participants were excluded from the early and late conditions respectively, as their PM scores were below chance. As in Experiment 1, trials with an RT of less than 150 ms or an RT of more than 4 SD above a participant's mean RT were removed from further analysis. Participants were removed if more than 10% of their responses exceeded the RT criterions. Based on this criterion, an additional four participants were removed from the early condition, and five from the late condition. The mean PM scores for the remaining participants in the early and late conditions were 0.64 ($SD = 0.27$) and 0.65 ($SD = 0.22$). The mean scores for the early ($t(11) = 8.18, p < .001$) and late ($t(9) = 9.33, p < .001$) conditions were significantly greater than chance. One additional participant was removed from the late condition for the slope plots and for the stimulus check. After removing trials with extreme reactions times, this participant had no responses for two items in the final block, thus response slopes and PM scores could not be calculated for the final block. This participant was retained for all other analyses (unless otherwise specified).

As in Experiment 1, PM scores were next calculated for each of the 18 blocks. A three-way between-within ANOVA (Condition × Stimulus type × Block) was run with these PM scores as the dependent variable. The ANOVA uncovered no significant main effect of stimulus type, $F(1, 17) = 0.20, p > .1$. Data were collapsed across stimulus type for the remaining analysis.

Response slopes were computed in the same manner as for Experiment 1, and are shown in Figure 4. In both conditions, participants adapted to the shift. There was, however, notably little change in responding after the first partial shift in training probabilities (i.e., in Blocks 7 and 8 in the early condition and in Blocks 11 and 12 in the late condition). Participants appeared to adapt more to the second shift, with mean slopes crossing through chance responding by Blocks 12 and 15 for the early and late conditions respectively.

Participants appeared to learn more slowly after the shift than they did during initial learning. The contrast in learning speeds between the pre- and post-shift learning was more evident than in Experiment 1. In both conditions, pre-shift response slopes were above the objective response slopes by Block 1. However, after the shift, the mean response slopes do reach the objective training probabilities in either conditions.

A two-way between-within subjects ANOVA was conducted on the slope estimates. The between-subjects factor was condition (early vs. late). The within-subjects factor was blocks. Due to the gradual-shift design of the present experiment, the training-block factor had 8 levels: these ranged from the two final pre-shift blocks to the second block following the final shift. Thus, for the early condition, the 8 blocks were Blocks 4 to 12, and for the late condition, they were Blocks 8 to 16. A Huynh-Feldt correction was used to correct for violations of sphericity. The change in slope across blocks was significant,

$F(3.72, 74.39) = 23.35, p < .001, \eta_p^2 = .54$, but there was no significant effect of condition, $F(1, 20) = 0.57, p > .1$, and no interaction, $F(3.72, 74.39) = 0.75, p > .1$.

A decisional-recency analysis was conducted as in Experiment 1. The results of this analysis are shown in Figure 5, with the upper and lower panels showing the P(shift) for identical and maximally different pairs of stimuli, respectively. As for the first study, data were simultaneously aligned on the first block and the block at which the training probabilities shifted; that is, Block 6 for the early condition and Block 10 for the late condition. As in Experiment 1, the P(shift) can be seen to decrease for identical items and increase for the maximally dissimilar items with training.

Statistical confirmation of the apparent error discounting revealed by the decisional-recency analysis in both Experiments 1 and 2 was provided by a multilevel generalized linear model (Baayen, Davidson, & Bates, 2007). To maximize power of the analysis, the mid and late conditions of Experiment 1 were analyzed together with the mid and late condition from Experiment 2. The early condition of Experiment 1 was excluded from the analysis as it had only one pre-shift block. Because the decisional-recency analysis pointed to the gradual discounting of errors *before* reinforcement probabilities were shifted, only pre-shift blocks were considered for each of the conditions. Thus, from Experiment 1, the analysis included Blocks 1 to 10 and 1 to 15 for the mid and late conditions respectively. From Experiment 2, the analysis included Blocks 1 to 6 and 1 to 10 for the early and late conditions respectively. To minimize the influence of the less diagnostic trial-pair transitions involving moderately dissimilar pairs of stimuli, only perceptual differences 0 (identity) and 3 (maximally dissimilar) were considered. The dependent variable was whether or not the response on the current trial differed from the preceding response (coded as 1 or 0). The perceptual difference between the two consecutive stimuli (0 or 3) was entered as a fixed-effect factor and was fully crossed with block and with a factor called 'error-before' which captured whether or not an error had been made on the previous trial.

The results are summarized in Table 1 and confirm the pattern in Figures 3 and 5. We highlight four effects: (a) The greater the perceptual difference between two successive stimuli, the more likely people were to switch their response category. (b) An error was more likely to be followed by a switch than a correct response. (c) Those two variables interacted such that a switch became less likely with increasing perceptual difference if an error had just been committed (compare the two panels of Figure 3 for a particularly clear illustration of this interaction). (d) Crucially, the role of an error declined with blocks (EB $\times$ block interaction), thus attesting to the presence of error discounting.

In summary, the trial-by-trial contingency data from both experiments replicated the decisional recency previously reported by Jones et al. (2006). We additionally established the presence of error discounting: People gradually adjusted the probability with which they shifted their response on trials following errors.

Computational Modeling

Both experiments provided multiple indications for the presence of error discounting. First, there was evidence for slower post-shift learning than during the initial learning; this was particularly evident in Experiment 2. Second, in Experiment 2, there was a notable delay in adaptation to the new reinforcement probabilities. Third, the decisional-recency analysis showed that at a trial-by-trial level, people gradually came to ignore errors as training progressed, even before reinforcement probabilities were shifted. We next turn to quantitative modeling to investigate further the existence of error discounting. The aim of using quantitative modeling was twofold. First, to verify whether an error-discounting mechanism could improve the ability of computational models to account for the data. Second, to pursue a theoretical exploration of the processes underlying error discounting.

We applied four models to the data. One model, the GCM (Nosofsky, 1986), was used to explore a potential alternative explanation for error discounting based on sample size. The other three models were the MAC model (Stewart et al., 2002), RASHNL (Kruschke & Johansen, 1999), and ATRIUM (Erickson & Kruschke, 1998). In each of the latter three models we instantiated a computational error-discounting mechanism. The aim of including an error-discounting mechanism was to determine whether, after controlling for the number of parameters, theories of categorization with an error-discounting mechanism were able to better account for the data than theories without error discounting. We give a brief summary of each model and the modeling results below. Full details of the models, model fitting procedures, and model performance can be found in the Appendix.

GCM. The GCM is an exemplar-based model that stores representations of all previously encountered stimuli. The model categorizes new stimuli by making a similarity comparison between the new stimulus and all stored exemplar representations. The GCM was used to explore an alternative explanation for error discounting based on sample size. It is possible that shifts in the feedback probabilities might have a small impact on behavior simply due to the large number of stored exemplars consistent with the initial training probabilities which "drown out" the later (and fewer) exemplars stored with the revised training probabilities. Due to its exemplar-based architecture, the GCM was a fitting choice to explore this sample-size explanation. The GCM contains two free parameters. The first is a specificity parameter, c. The specificity parameter controls the rate at which the psychological similarity to a stored exemplar decreases with increasing perceptual distance from a given stimulus. The second parameter is a scaling parameter, γ, that controls the decisional criterion of the model. The γ parameter can control whether the model responds at chance, probability matches, or maximizes.

MAC. Unlike most of its rivals, the MAC model uses only feedback and perceptual information from the previous trial (or limited set of trials) to make a category decisions. If the previous item was categorized correctly, then the MAC is more likely to give the same response to a perceptually similar item on the following trial. Conversely, the model is more likely to give an alternative response to a perceptually different item. This utilization of information from the immediately preceding trial(s) only makes the MAC model a prime candidate for exploring the impact of error discounting at the level of trial-by-trial decisional recency. Here we use the original MAC (Stewart et al., 2002), as opposed to an extended version of the model (Stewart & Brown, 2004). As noted by Stewart and Brown (2004), with greater trial memory, the MAC approaches the behavior of the GCM. Thus, the original MAC (with a single-trial memory) can provide greater comparative contrast to the GCM than can the extended MAC.

The MAC has a single parameter, b, which controls the rate at which the psychological similarity to the previous stimuli decreases with greater perceptual similarity between the current and previous items. We instantiated an error-discounting mechanism in the MAC by gradually attenuating the role of the previous stimulus for trials following an error. Specifically, we used two separate values of b following correct and following incorrect responses, respectively, and the value of b following incorrect responses was reduced by a factor r on each trial, given by:

$$r = 1/(1 + \rho), \tag{2}$$

where ρ was a free parameter that controlled the rate at which the similarity parameter, b, was reduced.

RASHNL. RASHNL is an exemplar-based model of associative learning. The model is an extension of the exemplar model ALCOVE (Kruschke, 1992). Like the GCM, RASHNL makes categorization decisions on the basis of a similarity comparison between the present stimulus and all stored exemplar stimuli. Unlike the GCM, RASHNL includes an error-based, associative-learning mechanism that controls the strength of association weights between exemplars and categories. The standard version of RASHNL can also learn to distribute its attention across stimulus dimensions. Due to the use of a single-dimension task, the attention features of RASHNL were not required for the present simulations.

RASHNL was chosen for its built-in error-discounting mechanism. The mechanism in RASHNL works by decreasing, or "annealing," the rate at which the model learns the associations between exemplars and categories. On each trial, an association learning rate, λ, is multiplied by the factor r, given in (2). As before, ρ is a free parameter controlling the rate at which the learning rate is reduced across trials. Higher value of ρ lead to fast annealing. As RASHNL uses errors to learn, by annealing the learning rates, as training progresses, RASHNL effectively ignores errors. RASHNL has two additional parameters, c, and φ, that are equivalent to the GCM's specificity parameter, c, and the decision criterion parameter, γ, as outlined above.

rATRIUM. To extend the potential generality of our conclusions beyond exemplar- and recency-based models, we also included a rule-based variant of ATRIUM (Erickson & Kruschke, 1998). At present both rule and exemplar models are popular within the probabilistic categorization literature, with prior research providing conflicting support for each type of model (Juslin, Jones, Olsson, & Winman, 2003; Kalish & Kruschke, 1997; McKinley & Nosofsky, 1995; Rouder & Ratcliff, 2004). As the present study is not designed to distinguish between these models types, we included a rule-based model to ensure that any results were not specific to a particular type of theory. The full version of ATRIUM is a hybrid rule and exemplar model. For present purposes we used only the rule module, which we term, rATRIUM. rATRIUM categorizes items by dividing the category space by a rule boundary. Each side of this boundary is associated with a particular category. The further a new item falls on one side of the boundary, the more likely it is that the item will be placed in the associated category. Like RASHNL, rATRIUM has an associative-learning mechanism. This associative learning mechanism allows rATRIUM to learn associations between categories and each side of the category boundary. To explore error discounting, we implemented the same error-discounting mechanism in rATRIUM as in RASHNL. As in RASHNL, rATRIUM's learning-rate parameter, λ, was annealed across training. The parameter, λ, controls the rate at which rATRIUM learns associations between categories and each side of the category boundary. The annealing rate was controlled by the free parameter ρ, as given in (2). rATRIUM includes two additional free parameters, a noise parameter, μ, which controls the standard deviation of perceptual noise around the rule boundary. Greater values of μ, decrease the ability of the model to distinguish an item's location relative to the rule boundary. The final parameter, φ, is a decision criterion parameter, equivalent to the γ and φ parameters in the GCM and RASHNL, respectively.

Model Fitting Procedures and Modeling Results

For the three models with an error-discounting mechanism, the MAC, RASHNL, and rATRIUM, we ran two sets of fits, one with the error-discounting mechanism functioning, and one with it switched off. The models were fit at the level of individuals by maximizing the binomial log-likelihood.

Table 2 summarizes the fit statistics (i.e., negative log-likelihoods, Akaike information criterion, AIC_c, and Bayesian information criterion, BIC) for each of the models (figures of the model predictions can be found in the Appendix). From the table, the GCM provided a poor account of the results, with relatively high AIC_c and BIC values than compared to RASHNL and rATRIUM. From inspection of the model predictions, the GCM was unable to adapt to the shift as fast as participants did in all conditions, with the exception of the early condition of Experiment 1. The poor performance of GCM rules out the sample-size explanation of error discounting.

RASHNL, with annealing on, provided the best account of the data. Inspection of the model predictions showed that RASHNL provided a good quantitative fit of the data, closely capturing the participant's behavior throughout the shift in all conditions. The fits of RASHNL with annealing on were followed by rATRIUM with annealing on, and then RASHNL and rATRIUM with annealing off. The MAC did not provide a good quantitative fit; however, inspection of the best-fitting predictions of the model showed that the MAC qualitatively captured the data, including the shift in reinforcement.

Crucially, for all three models with an error-discounting mechanism, the fits were better when the mechanism was turned on than when it was off. For the MAC, RASHNL, and rATRIUM, lower AIC_c and BIC values were obtained when annealing was on than when annealing was off. Likelihood-ratio tests confirmed that each of the models fit significantly better when they included an annealing mechanism (RASHNL: $\chi^2(65) = 1034.81, p < .001$, rATRIUM: $\chi^2(65) = 1313.56, p < .001$, MAC: $\chi^2(64) = 2547.14, p < .001$). The fact that all three models fit better with error discounting suggests that (a) error discounting is required to accurately capture probabilistic categorization behavior, and (b) that this holds true regardless of the particular type of category representation.

General Discussion

Summary of Findings

The present article sought to investigate (a) whether people gradually tend to discount errors during probabilistic categorization, and (b) what the underlying mechanism of error discounting are, if present. Taken together, the results clearly reveal the presence of error discounting during probabilistic category learning. Error discounting can be inferred from (a) the difference between the initial fast learning and the slower post-shift learning, (b) the apparent lack of adaptation to the first partial shift in Experiment 2, (c) the decline in likelihood with which participants shifted their category response on trials following an error, and (d) the fact that both associative-learning models and the MAC model handled the data significantly better when they included an error-discounting mechanism than when they did not.

The present evidence for error discounting is generally consistent with previous findings (Edgell, 1983; Edgell & Morrissey, 1987) that implicated the presence of annealing in models of categorization (Kruschke & Johansen, 1999). Unlike those precedents, however, the unidimensional design of the present studies precluded alternative interpretations based on blocking. Before we discuss the implications of our results, we briefly address some possible limitations.

Limitations and Concerns

Participants in our studies were not informed about a possible change in the probabilistic environment. In the natural world, by contrast, people may be more wary of changing probabilities and might thus adapt faster than observed here. However, the fact that learning in Experiment 2 continued to be slow after participants had transited through chance responding—at which point they would likely be aware of the changed reinforcement probabilities—suggests that awareness of a shift might not prevent (or undo) error discounting. Moreover, the decisional-recency analysis that showed a decline in the probability of shifting responses following errors, was run on pre-shift blocks only. Thus, people were shown to discount errors even before any shift had taken place. The shift in reinforcement probabilities thus helped highlight the existence of error discounting; however, it was not critical for error discounting to occur and to be detected.

Critics might also puzzle why the extent of pre-shift training failed to have any effect. If error discounting is gradual, why did people's adaptation rate not decrease with more pre-shift training? Our response is twofold: First, given the relative simplicity of our task, even limited training may have been sufficient to induce at least some discounting. This possibility is supported by the finding that even after the very first block of training, performance was far above chance and probability matching close to the objective target. It is also supported by the visible trends in the decisional-recency analysis. Second, RASHNL and rATRIUM required annealing for their account of the data, irrespective of the extent of pre-shift training and even when parameter values were constrained to be identical across conditions. This finding confirms that error discounting commenced early on during training and then continued throughout the experiment.

Turning to potential theoretical limitations, we note that the use of a single stimulus dimension—while eliminating an alternative blocking explanation—rendered the attention and gain features of RASHNL and rATRIUM irrelevant. As it is assumed in RASHNL that the attention and gain learning rates are also annealed (Kruschke & Johansen, 1999), the role of these features must therefore await further exploration. Likewise, it remains unclear how discounting operates in multi-dimensional environments. On the one hand, error discounting may play a greater role in complex environments, because if people have more difficulty determining the underlying probability structure, they may show a greater tendency to ignore errors. On the other hand, it may be that with multi-dimensional stimuli, error discounting is overshadowed by blocking effects, and thus has little impact on its own.

Finally, error discounting might interact with other effects and strategies present in multi-dimensional probabilistic categorization. For instance, Little and Lewandowsky (2009a) showed that compared to a deterministic condition, participants learning an identical category structure under probabilistic reinforcement spread their attention across a wider selection of possible cues. Specifically, people detected a non-diagnostic correlation among cues that escaped notice under deterministic learning conditions. It follows that in multi-dimensional categorization, error discounting might only have an impact once alternative strategies (i.e., attentional allocations) have been exhausted. Those issues await resolution by future research.

Theoretical Implications

In the present study we used three types of categorization models; an exemplar model, a rule-based model, and a recency-based model, to explore the existence of error discounting. The present study was not designed to distinguish between these types of categorization models, nor was it designed to determine how people represent categories. Instead, the inclusion of the three model types demonstrated that regardless of how categories are represented, error discounting is important to account for probabilistic categorization behavior. We therefore conclude that annealing of learning rates and error discounting deserve a prominent theoretical role in approaches to probabilistic categorization.

If we accept the necessity for error discounting, how is it best described at a conceptual level? Does error discounting reflect an on-going and ever-increasing habituation to a constant error signal? Alternatively, does error discounting reach a plateau, thus effectively differentiating between two error signals—one that is unavoidable due to the probabilistic environment and (potentially) another one that is unexpected and arises from changes in the environment?

The former alternative corresponds to the instantiation of annealing in RASHNL and rATRIUM, whose learning rates ultimately and inevitably reach 0 (whereupon no further adaptation to a shift occurs, as confirmed by simulations involving a large amount of additional pre-shift training that are not reported in detail here). If people, as assumed by the models, progressively discount more and more error, how would they notice a gradual shift in reinforcement probabilities (as in our Experiment 2), and how would they learn from it? One possibility is that not all feedback-based learning is error driven. Indeed, there is recent neuroscientific (Histed, Pasupathy, & Miller, 2009) and behavioral (Smith & Kimball, 2010) evidence that people are capable of learning from feedback on correct responses. These recent findings run contrary to the common error-focused assumptions of connectionist models (e.g., Gluck & Bower, 1988a, 1988b; Rumelhart et al., 1986). We are not aware of any connectionist learning that is driven by *correct* responses, although these recent results (Histed et al., 2009; Smith & Kimball, 2010) may stimulate developments in that direction.

The latter alternative, namely that error-discounting reaches a plateau, is not currently instantiated in RASHNL and rATRIUM but nonetheless seems rather attractive for two reasons: First, people in both experiments *were* responsive to shifts, albeit sometimes with a considerable delay (in Experiment 2). Second, the extent of pre-shift training did not affect people's responsiveness to the shift in either experiment. This result is difficult to reconcile with a notion of growing habituation to error and it is more compatible with the idea that a *certain amount* of error is discounted, but that any further error is used for associative learning.

The idea that people discount some variability in the error signal resonates with the idea that people adjust their strategies avoid *over-fitting* or, in other words, overcompensating for noise (Myung & Pitt, 2004). The idea of over-fitting is germane if we assume that participants are in effect trying to distinguish some true feedback signal from variation or noise within that signal. Hence, participants might be aware that the overall environment has changed but are unaware of the extent of that change. Error discounting can therefore be viewed as a rational way to exclude excess variation from influencing learning. This is similar to modern machine learning algorithms which typically include a regularizing term (i.e., a penalty on the fitting measurement) to effectively limit the types of patterns that can be learned and reduce over-fitting (Bishop, 2006). Other approaches, such as the Bayesian learning approach we consider below (see e.g., Bishop, 2006, Chapter 3), implement regularization through use of prior probability distributions over relevant patterns.

The observation that error discounting exists at the level of decisional-recency flags the importance of considering corrective feedback when exploring sequential effects in categorization. Previous analyzes of decisional-recency effects (e.g., Jones et al., 2006; Stewart et al., 2002) have focused on the impact of category-label *feedback* irrespective of whether or not the previous response was an error. As we have observed, people react differently following errors and correct responses; additionally, people adjust their behavior in response to errors over time. These reactions to errors in categorization suggest that it is important to consider corrective feedback when exploring decisional-recency effects.

So far, the focus here has been on participants' tendency to ignore error-based corrective feedback information. While participants did tend to discount errors, and thus ignored feedback, participants simultaneously showed signs of precise utilization of feedback information, as evidenced by people's highly-accurate probability-matching behavior. In the present tasks, the probability with which people categorized items approximated the probability with which the items belonged to each category. This accurate probability matching cannot arise without accurately observing the feedback information. Thus, participants simultaneously exhibited precise utilization of feedback information and a tendency to ignore feedback information. This duality of the utilization of feedback information is counter-intuitive: How can people dismiss feedback while being demonstrably attuned to it at the same time? The answer is implied by our modeling: The probability-matching behavior is established early on, when associations between stimuli and outcomes are learned on the basis of feedback. Once established, those associations persist (unless a reversal shift takes place) despite the increasing discounting of errors and the associated feedback.

Links to Bayesian Perspectives on Category Learning

The recent popularity of probabilistic-descriptive approaches to cognition (e.g., Chater, Tenenbaum, & Yuille, 2006) raises the possibility that error discounting might be tractable within a Bayesian approach. Bayesian approaches to cognition take into account our beliefs and expectations in the form of a prior probability distribution over potential hypotheses. Our prior beliefs are transformed into *posterior* beliefs in light of new data; hypotheses which are consistent with the data are maintained whereas inconsistent hypotheses are discarded. In our task, as knowledge accumulates about the probability with which each item falls into each category, the distribution that represents those beliefs becomes more peaked and less variable with continued training. Hence, the influence of each individual stimulus on the posterior expectations of an item's category membership decreases: Less variable prior distributions require more new evidence (i.e., higher error rates) in order for new hypotheses to have any weight in the posterior. Error discounting may thus be interpretable in Bayesian terms as reflecting the continued "sharpening" of the distribution over the reinforcement rate of each item.

Implications of Error Discounting

Although our work was confined to probabilistic categorization, we can draw some connections to other situations involving decision making and industrial processes.

One perspective on error-discounting is that people minimize cognitive "expenditure" by dismissing the feedback signal that would otherwise entail further learning. This reduction in cognitive expenditure is also at the heart of the simple heuristics that underlie much of human decision making, and which have been characterized as maximizing our return on the investment of cognitive effort (Gigerenzer, Todd, & ABC Research Group, 1999). Similar to decision making heuristics—such as the recognition heuristic (Goldstein & Gigerenzer, 1999, 2002)—error discounting is highly adaptive for environments with unavoidable error, in which continuing to attend to errors entails no benefit for accuracy, whereas ignoring errors may free up limited cognitive resources.

However, in the same way that decision-making heuristics may incur a cost when the environment fails to conform to people's expectations (Tversky & Kahneman, 1974), error discounting can also incur a high cost in some environments. For example, pilots who ignore frequent warnings from an autopilot system are at increased risk of causing an accident (Farrell & Lewandowsky, 2000; Parasuraman & Riley, 1997; see also, Moray2007). High false alarm rates for automated error detection systems result in people slowing their response to, or simply ignoring warnings about potential errors (Parasuraman & Riley, 1997). For example, in a task monitoring the navigation, speed, power and cargo systems of a ship, if the system sets off frequent false error alarms, participants eventually ignore these warnings signals (Kerstholt & Passenier, 2000). With frequent false alarms, participants made fewer attempts to gain information needed to asses the problem, and were slower to access this information when they did. This behavior constitutes a good analog to the error discounting observed in our studies. However, in many real-life situations, poor reliability of automated systems interacts with the complexity of those systems, thus arguably engendering error discounting in precisely those situations in which it is least advisable. Future work on trust in automation (Lewandowsky, Mundy, & Tan, 2000) is necessary to delineate the circumstances in which such inadvisable error discounting may occur.

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Appendix

Supplementary Material: Additional Computational Modeling Details and Additional Results

Craig, Lewandowsky, and Little (submitted) explored the existence of error discounting in probabilistic categorization. Part of their analysis involved the use a series of computational models, the GCM (Nosofsky, 1986), MAC (Stewart et al., 2002), RASHNL (Kruschke & Johansen, 1999), and ATRIUM (Erickson & Kruschke, 1998). Craig et al. (submitted) found that the MAC, RASHNL, and ATRIUM fit data from two probabilistic categorization experiments significantly better when they included a mechanism to discount errors. The GCM did not include an error-discounting mechanism, instead poor performance of GCM ruled out an alternative sample-size explanation for error discounting. This supplement provides additional details of the models, modeling procedures, and results from the fits with each of the four models.

GCM: A Sample-Size Explanation

A possible explanation for the experimental results in Craig et al. (submitted) was that the slow post-shift learning was not due to a discounting of error, but instead was an automatic by-product of the inevitable increase in memorized "sample size" during learning. Specifically, in the very early stages of training, when few items had been presented, on an exemplar view each further individual stimulus will have a relatively large impact. However, at later stages of learning, new items become increasingly insignificant in relation to the overall number of exemplars already encountered and memorized, thus limiting their impact. The slow adaptation to the shift in the experiments could therefore result from the small impact of items later in training relative to items early in training. We explored this alternative within the GCM (Nosofsky, 1986). The GCM contains no associative learning mechanism but represents all encountered instances in memory, thus providing a quantitative instantiation of the sample-size hypothesis.

GCM Specification. The GCM assumes that on each trial the current item activates all previously encountered stimuli stored in memory according to:

$$s_{ij} = exp(-c \times d_{ij}), \tag{A1}$$

where the similarity, s_{ij}, between items i and j is determined by the distance between them in psychological space, $d_{ij} = |x_i - x_j|$, which in the present case involves only a single dimension. (Note that for simplicity of exposition, all equations reported in this supplement are tailored to the fact that our stimuli were uni-dimensional.) The specificity parameter, c, determines the sharpness of the exponential function.

Similarities are converted to response probabilities by applying Luce's choice rule (Luce, 1963):

$$P(A|i) = \frac{(\sum_{j \in A} s_{ij})^{\gamma}}{(\sum_{j \in A} s_{ij})^{\gamma} + (\sum_{j \in B} s_{ij})^{\gamma})}, \tag{A2}$$

where the response scaling parameter, γ, allows responding to vary between probability-matching when $\gamma \simeq 1$ and maximizing when $\gamma \gg 1$ (Ashby & Maddox, 1993; Nosofsky & Johansen, 2000). Thus, upon presentation of a test stimulus, it is compared against all stored exemplars in each of the categories separately, and a response is selected based on which category yields the greatest summed similarity.

GCM Simulations. For the square stimuli used in the present study, the perceived psychological distance between adjacent stimuli has been shown to be equivalent to the actual perceptual distance between the stimuli (Colreavy & Lewandowsky, 2008). Thus, for the simulations, the four stimuli were coded as the integers 1 to 4. The GCM was fit separately to each participant's mean response probabilities for all items in all blocks. The

GCM was presented with the training sequences shown to participants. Parameters (c and γ) were capped at 25. Both parameters were estimated using the SIMPLEX algorithm (Nelder & Mead, 1965) to minimize the negative binomial log-likelihood:

$$-\ln L = -\sum_i d_i \ln(p_i) + (n_i - d_i)\ln(1 - p_i), \tag{A3}$$

where p_i is the model's predicted probability of category A for item i, d_i is the observed number of A responses made for item i, and n_i is the number of times item i was presented.

Table A1 shows the GCM's estimated parameters for both experiments. As shown in Figures A1 and A2, the GCM failed to capture the data in some crucial ways: In particular, it was unable to adjust its predictions in response to the probability shift. The GCM only managed to capture the early condition of Experiment 1, presumably because only 10 presentations of each item had occurred before the shift. However, in all other conditions, the GCM could not accumulate enough new evidence within the number of training trials to reverse its predicted probabilities. (Note, however, that there is a downward trend in the predicted slopes after the shifts, which indicates that given massively extended training, the GCM's performance might come to mirror the final outcome probabilities). In conclusion, the experimental results in Craig et al. (submitted) cannot be accommodated by an explanation based solely on memorized sample size.

MAC model: Error discounting and Decisional Recency

The MAC model (Stewart et al., 2002) makes categorization decisions based only on perceptual and feedback information provided on the previous trial. With the exception of the immediately preceding item, the MAC model does not retain a representation of previously encountered exemplars.

Formal description of the MAC model. We present two versions of the MAC model. The first is the standard version of the model as outlined by Stewart et al. (2002). The second is a new version of the model, modified to incorporate error discounting via an annealing mechanism.

The MAC model makes categorization decisions by computing the psychological distance between the current stimulus and the previous one (this distance is isomorphic to the perceptual difference used in our decisional-recency analysis). The probability of repeating the same category response, P(same), is given by:

$$P(same\ category) = e^{-bd^2} \tag{A4}$$

where d is the psychological distance between the current and previous stimulus, and b is a constant controlling the steepness of the Gaussian generalization gradient. Thus, b controls the rate at which psychological distance required to respond with a different category than the previous trial. Lower values of b will result in stimuli being perceived as more similar, thus increasing the probability that the response on the current trial will match the category feedback given on the previous trial.

The second, modified version of the MAC model includes an error-discounting mechanism, implemented by annealing the parameter b for trials following an error. Specifically, b was set to decrease across trials in this version, thus forcing the model to treat all items as if they were similar to the one just seen on the previous trial. Thus, the model effectively ignores the feedback from trials following errors.

To incorporate the error-discounting mechanism, we generated two forms of Equation A4. The first form of the equation was used on trials for which the response given on the previous trial was correct. The second version of the equation was used on trials following an error and instantiated annealing using the formalism provided in

RASHNL (Kruschke & Johansen, 1999). For this second form of the equation, the initial value of b was multiplied by an annealing factor, r, given by:

$$r(t) = 1/(1 + \rho \times t), \tag{A5}$$

where ρ is a freely estimated, non-negative, scheduling parameter that controls the rate of annealing. Larger values of ρ lead to faster annealing, and when ρ is clamped at zero, the model exhibits no annealing at all (and saves a parameter in the process). This annealing mechanism has the effect of reducing the steepness of the Gaussian generalization gradient on the psychological distance between pairs of stimuli over trials. Thus, the mechanism gradually reduces the probability with which the model will shift its category response on a trial following an error. In summary, the standard MAC model has one parameter, the size parameter, b. The modified MAC model includes both the size parameter, b, and the annealing parameter, ρ.

MAC model simulations. The MAC model was fit in the same manner as the GCM. For the modified version, the ρ parameter was capped at 100. In order to closely model the participants' trial-by-trial behavior, we used each participant's individual responses, rather than the MAC model's own responses, to guide the feedback given to the model. That is, for each response, the MAC model computed its prediction based on the participant's response and feedback on the previous trial, rather than its own preceding response (use of the model's own response considerably worsened the model's fit). One of the participants for the late condition of Experiment 2 was not fit, as after removing trials with extreme reaction times, this participant did not have responses remaining for all items across all blocks. The best-fitting parameter values are presented in Table A2.

The MAC model captured the data qualitatively, but not quantitatively; the fits to Experiments 1 and 2 are shown in Figures A3 and A4, respectively. The slope increased above chance early in training, and crossed through chance following the shift in reinforcement probabilities. However, the model's slopes fell short of participants' slopes and the model adapted to the shift much faster than did participants.

We do not consider the failure of the MAC model to quantitatively capture the data to be a core issue: The initial version of the MAC model was not designed to be a complete categorization model, but rather a means to explore perceptual recency effects in categorization Stewart et al. (2002). In that spirit, interest was on whether the model could better account for the data with the inclusion of an error-discounting mechanism.

RASHNL: Error Discounting via Annealing of Learning

Formal description of RASHNL. RASHNL (Kruschke & Johansen, 1999) is an exemplar-based connectionist model of probabilistic categorization that was developed as an extension of ALCOVE (Attention Learning COVEring map; Kruschke, 1992), which is itself an extension of the GCM (Nosofsky, 1986). Central to RASHNL is the concept of annealing of learning rates. This annealing mechanism captures error discounting by gradually decreasing the rate at which the model learns over time.

The annealed learning provides the model with various advantages over a fixed learning rate: In addition to the general notion that it allows for quick adaptation early in training, followed by slow fine-tuning of probabilistic responding (Amari, 1967; Heskes & Kappen, 1991; Murata et al., 2002), annealed learning may also help RASHNL avoid unduly high sensitivity to order among stimuli late in training, such as observed in ALCOVE (Lewandowsky, 1995). In confirmation, the limited tests of RASHNL available to date have consistently found that the annealing-based error-discounting mechanism improves the ability of the model to account for both probabilistic (Kruschke & Johansen, 1999) and non-probabilistic (Blair & Homa, 2005) categorization behavior.

RASHNL has a layer of input nodes that correspond to the dimensions of the stimulus. As RASHNL is an exemplar-based model, new items are categorized based on their similarity to previously encountered category members (Kruschke & Johansen, 1999; Nosofsky, 1986). Hence, the input nodes connect to a layer of hidden exemplar nodes, which correspond to the training stimuli. Activation of the jth exemplar node is given by:

$$h_j = \exp(-c \times |\psi_j - d|), \tag{A6}$$

where c, the specificity, is a free parameter that determines the slope of the gradient of the receptive field of each exemplar; that is, the slope of the exponential decline in similarity with increasing distance between the current stimulus and the jth stored exemplar. (The present stimuli were uni-dimensional; accordingly, we removed RASHNL's gain activation, attention shifting, and attention-updating mechanisms, all of which only apply to multi-dimensional stimuli.)

Exemplar nodes connect to output nodes, which correspond to the available categories. Activation of the kth category node, a_k, is given by:

$$a_k = \sum_j w_{kj} \, h_j, \tag{A7}$$

where w_{kj} is a weight associating an exemplar with a category. Category activations are mapped onto response probabilities using a version of Luce's (1963) choice rule, such that the probability of categorizing a stimulus into category K is determined by the exponentiated activation of category K over the sum of the exponentiated activation of all categories, given by:

$$P(K) = \exp(\varphi a_k) / \sum_i \exp(\varphi a_i), \tag{A8}$$

where φ is a scaling parameter representing decisiveness. If φ is large, then a small activation advantage for category K will result in large preference for category K, corresponding to maximizing behavior. Conversely, if φ is small, the response will be more uncertain and in proportion to the relative activations, thus corresponding to probability matching.

RASHNL is an error-driven learning model, such that each response is followed by feedback indicating the correct category in the form of teacher values for each category node. The error associated with a response is given by:

$$E = \frac{1}{2} \sum_k (t_k - a_k)^2, \tag{A9}$$

where t is the teacher value, such that $t_k = 1$ if the stimulus is a member of category k, and $t_k = 0$ if the stimulus is not a member of category k.

Learning proceeds through the minimization of E via adjustment of association weights by gradient descent on error, given by:

$$\Delta w_{kj} = \lambda (t_k - a_k) h_j, \tag{A10}$$

where λ represents the learning rate. Crucially for the current experiments, this learning rate is annealed, such that as training progresses the rate of learning is slowed. Although several annealing mechanisms have been explored within the neural network literature (Amari, 1967; Bös & Amari, 1998; Heskes & Kappen, 1991; Müller, Ziehe, Murata, & Amari, 1998; Murata et al., 2002), RASHNL uses a "search and converge" mechanism to decrease learning rates (see e.g., Darken & Moody, 1991). On each trial, t, the initial learning rate is multiplied by an annealing factor, r, as per the earlier Equation A5. The annealing function allows the model to make large shifts in learning early in training, whereas from around trial $1/\rho$ onward, the learning rates rapidly reduce and converge to zero. In addition to the annealing rate, ρ, this version of RASHNL had three other free parameters: specificity, c, the probability-mapping parameter, φ, and the weight-learning rate, λ.

RASHNL simulations. The model was applied to the data in the same manner as the GCM. RASHNL was fit using the models' own corrective feedback to guide model behavior. In both fits, we compared two versions of RASHNL: One in which the annealing parameter, ρ , was freely estimated and another one in which it was set to zero.

Figures A5 and A6 show the mean predictions of RASHNL when fit to the data of individual participants with annealing turned on for Experiments 1 and 2 respectively. Quite in contrast to the GCM and the MAC model, RASHNL captured the fast initial learning and slower post-shift learning that was displayed by participants. The corresponding mean (and median) best-fitting parameter values (aggregating across the fits to individual subjects) are shown in Table A3.

rATRIUM: Annealing Without Exemplars

The majority of rule-based models, including the GRT, do not include associative learning mechanisms. As an associative learning mechanism is particularly suited for investigating error discounting, we selected the rule module of ATRIUM (Erickson & Kruschke, 1998) as an alternative candidate model for the present data. ATRIUM's rule module learns to associate rules with particular categories via a standard network learning algorithm, permitting implementation of annealing in the same manner as in RASHNL.

ATRIUM is a hybrid model that relies on both exemplars and rules; here, we eliminated the exemplar module because we were exclusively interested in the generality of annealing and its applicability within a rule-based architecture (hence we use the label rATRIUM for this variant of the model from here on). rATRIUM divides the category space by a rule boundary set perpendicular to the relevant stimulus dimension. The stimulus dimension is represented by two rule nodes, r_{small} and r_{large}, whose activations are given by:

$$r_{small} = 1 - \frac{1}{1 + \exp[-\mu(d + \beta)]}, \tag{A11}$$

and by:

$$r_{large} = \frac{1}{1 + \exp[-\mu(d + \beta)]}, \tag{A12}$$

where d represents the value of a given item on the stimulus dimension. Each of these rule nodes forms a sigmoid threshold function, centered on the rule boundary, such that larger dimensional inputs will result in higher activation of the large rule node, while smaller dimensional inputs will result in a higher activation of the small rule node. The parameter μ represents the gain of the sigmoid (i.e., its steepness), and thus controls the level of perceptual noise (or its equivalent) as dimensional values approach the rule boundary. Large values of μ result in stimuli close to the rule boundary being more confusable. The parameter β controls the position of the rule boundary.

The rule nodes are connected to output nodes that correspond to the possible category selections. The activation of output nodes, a_k, for each category, k, is calculated as the sum of the activations of the small and large rule nodes, given by:

$$a_k = w_{k_{large}} r_{large} + w_{k_{small}} r_{small}, \tag{A13}$$

where the activation is moderated by the learned association weights, w_k, between the rule and output nodes. As in RASHNL, the association weights are updated by minimizing mean square error during learning:

$$\Delta w_{kj} = \lambda(t_k - a_k) r_j, \tag{A14}$$

where λ is a freely estimated parameter which controls the rate of learning. Finally, output activations are converted into probabilities as in Equation A8 in RASHNL.

For present purposes, the annealing mechanism from RASHNL, given in Equation A5, was imported into rATRIUM: Thus, an annealing rate, ρ, controlled the rate at which the weight learning rate, λ, was adjusted on successive trials.

In summary, the model has four free parameters: A gain constant, μ, which sets the standard deviation of the perceptual noise; a scaling constant, φ, which maps output probabilities to participant responses; the annealing rate, ρ; and learning rate, λ.

The model was fit in the same manor as the fits with RASHNL. Figures A7 and A8 show the fits of rATRIUM to Experiment 1 and Experiment 2 respectively. Like RASHNL, rATRIUM provided a good fit to the data. As shown in the figures, the model closely tracked the behavior of the participants throughout training.

Fit Statistics

In Craig et al. (submitted), a series of fit statistics were used to compare the fits of the four models, viz. AIC (Akaike Information Criterion; Akaike, 1974) and w_i(AIC) (AIC weights; Wagenmakers & Farrell, 2004), and BIC (Bayesian Information Criterion) and w_i(BIC) (BIC weights; Wagenmakers & Farrell, 2004) to compare the models. AIC and BIC adjust for model complexity and flexibility. When corrected for small sample sizes, the AIC (AIC$_c$) is given by:

$$AIC_c = -2\ln L + 2V + \frac{2V(V+1)}{n-V-1}, \tag{A15}$$

where L is the maximum likelihood for the given model with V free parameters taken over n observations. The corrected AIC is recommended for use of samples where the ratio of data points to parameters is less than 40. The AIC thus combines two sources of information: Lack of fit (represented by the log likelihood) and a penalty term for model complexity (represented by the second and third terms in the above equation). The BIC is given by:

$$BIC = -2\ln L + V\ln n. \tag{A16}$$

Unlike AIC, whose penalty relies on the number of parameters only, the BIC additionally penalizes models based on the number of data points being fitted.

AIC_c and BIC values were converted into AIC and BIC weights (Wagenmakers & Farrell, 2004). The $w_i(\text{AIC})$, and $w_i(\text{BIC})$, represent the conditional probabilities that the model M_i is the best of the set of models being compared.

In Craig et al. (submitted), a likelihood-ratio test (Lamberts, 1997) was used to determine whether the loss of fit associated with removal of error discounting, by setting ρ to zero, was statistically significant. The likelihood-ratio test is given by:

$$\chi^2 = -2[\ln L(restricted) - \ln L(general)], \tag{A17}$$

where $\ln L(general)$ is the log-likelihood of the version of a model that includes annealing (i.e., $\rho > 0$), whereas $\ln L(restricted)$ is the log-likelihood of the restricted version of a model, with annealing set to zero.

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Table 1

Results of multilevel generalized linear model analysis of trial-by-trial contingencies from the mid and late conditions of Experiment 1 and Experiment 2.

Fixed Effect[a]	Estimate	z-value	p
perceptual difference (PD)	4.21	15.31	$< .0001$
error before (EB)	2.53	12.60	$< .0001$
block[b]	0.01	0.58	n.s.
PD $\times$ EB	-3.92	-14.83	$< .0001$
PD $\times$ block	.02	0.79	n.s.
EB $\times$ block[c]	$-.07$	-3.33	$< .001$
PD $\times$ EB $\times$ block	.07	1.86	n.s.

[a]The best-fitting model ($BIC = 6693$) also included subject, EB, PD, and block as random effects.

[b]linear effect of block when the preceding response was correct ($EB = 0$)

[c]adjustment to linear effect of block when the preceding response was incorrect ($EB = 1$)

Table 2

Negative log-likelihood ($-\ln L$), corrected Akaike Information Criterion values, (AIC_c), AIC weights, w(AIC), Bayesian Information Criterion values, (BIC), and BIC weights, w(BIC), for fits of the GCM, the MAC model, RASHNL and rATRIUM across both experiments and all participants.

Model	Annealing	Free params	$-\ln L$	AIC_c	w(AIC)	BIC	w(BIC)
GCM	-	2	27295.92	54274.55	0	55147.81	0
MAC	On	2	27601.90	54880.43	0	55751.22	0
MAC	Off	1	28875.47	59067.51	0	58024.65	0
RASHNL	**On**	**4**	**25256.81**	**50315.53**	**1**	**51625.56**	**1**
RASHNL	Off	3	25774.22	51321.99	0	52382.39	0
rATRIUM	On	4	25637.00	51075.90	0	52385.92	0
rATRIUM	Off	3	26293.77	52361.10	0	53421.50	0

Note: Bold items indicate best fit.

Table A1

Median (Mdn), mean (M), and standard deviation (SD) of estimated parameter values across participants and negative log-likelihood values (-lnL) for the fits with GCM for each condition of both experiments.

Exp.	Cond.	Parameters						
		γ			c			$-\ln L$
		Mdn	*M*	*SD*	*Mdn*	*M*	*SD*	
1	Early	2.55	2.37	1.86	1.42	13.20	20.76	5939.48
	Mid	1.07	1.11	0.71	2.90	9.87	10.96	6163.92
	Late	1.19	1.74	1.34	1.43	5.46	8.81	5924.28
2	Early	1.55	1.57	1.04	2.09	9.23	11.30	5321.91
	Late	1.50	1.67	0.87	2.10	6.25	9.92	3946.33

Table A2

Median (Mdn), mean (M), and standard deviation (SD) of estimated parameter values across participants and negative log-likelihood values (-lnL) for the fits of the MAC model with annealing on and off for each condition of both experiments.

Exp.	Cond.	Parameters						
		ρ			b			$-\ln L$
		Mdn	*M*	*SD*	*Mdn*	*M*	*SD*	
1	Early	0.194	7.795	25.704	0.43	0.41	0.22	6425.07
	Mid	0.414	36.089	49.447	0.45	0.47	0.18	5963.39
	Late	0.497	14.751	36.122	0.39	0.54	0.51	6182.40
	Early	0.	0.	0.	0.35	0.35	0.23	6663.88
	Mid	0.	0.	0.	0.44	0.50	0.40	6171.74
	Late	0.	0.	0.	0.34	0.41	0.22	6420.13
2	Early	0.812	41.810	51.368	0.47	0.43	0.19	5337.18
	Late	1.004	13.287	32.690	0.46	0.51	0.13	3693.88
	Early	0.	0.	0.	0.38	0.44	0.35	5647.71
	Late	0.	0.	0.	0.40	0.44	0.15	3972.02

Table A3

Median (Mdn), mean (M), and standard deviation (SD) of estimated parameter values across participants and negative log-likelihood values (-lnL) for the fits with RASHNL with annealing free or set to zero for each condition of both experiments.

Exp.	Cond.	Parameters												
		ρ			φ			λ			c			$-\ln L$
		Mdn	*M*	*SD*	*Mdn*	*M*	*SD*	*Mdn*	*M*	*SD*	*Mdn*	*M*	*SD*	
1	Early	0.019	0.457	1.586	3.89	4.88	3.88	0.26	1.46	3.21	1.17	5.39	15.43	5688.31
	Mid	0.006	0.024	0.030	4.13	3.95	2.14	0.23	0.55	0.72	0.98	2.97	7.52	5303.15
	Late	0.032	0.054	0.088	2.37	3.03	1.79	0.78	1.66	2.55	1.57	8.48	13.19	5696.02
	Early	0.	0.	0.	4.05	10.91	24.97	0.08	0.35	0.58	1.42	1.42	0.91	5729.54
	Mid	0.	0.	0.	4.09	4.33	3.04	0.17	0.39	0.50	0.82	3.00	8.30	5442.77
	Late	0.	0.	0.	2.73	3.36	2.16	0.09	0.44	0.61	1.38	5.62	11.18	5764.01
2	Early	0.035	0.285	0.748	3.14	3.50	2.61	0.39	2.71	7.08	0.79	18.43	55.05	4937.05
	Mid	0.025	0.038	0.040	4.44	7.10	8.53	0.30	0.76	0.90	0.76	1.05	0.82	3632.29
	Early	0.	0.	0.	3.70	6.03	7.29	0.07	0.47	0.63	1.17	6.38	12.44	5115.43
	Late	0.	0.	0.	5.21	8.68	9.32	0.03	0.08	0.11	0.91	4.16	10.10	3722.47

Table A4

Median (Mdn), mean (M), and standard deviation (SD) of estimated parameter values across participants and negative log-likelihood values (-lnL) for the fits with r*ATRIUM with annealing free or set to zero for each condition of both experiments.*

Exp.	Cond.	Parameters												
		ρ			φ			λ			μ			$-\ln L$
		Mdn	*M*	*SD*	*Mdn*	*M*	*SD*	*Mdn*	*M*	*SD*	*Mdn*	*M*	*SD*	
1	Early	0.041	0.092	0.142	1.75	1.98	1.83	3.41	3.69	3.18	0.93	1.02	0.43	5769.22
	Mid	0.002	0.375	1.181	1.66	1.96	1.26	0.88	3.74	6.72	0.94	1.10	0.70	5375.43
	Late	0.011	0.049	0.118	1.27	8.65	26.31	1.20	2.06	2.25	0.74	0.95	0.65	5754.83
	Early	0.	0.	0.	1.76	2.08	1.80	0.41	0.64	0.83	1.01	1.07	0.66	5892.26
	Mid	0.	0.	0.	2.55	2.29	1.41	0.26	0.48	0.46	0.92	0.97	0.34	5484.35
	Late	0.	0.	0.	1.28	1.66	1.18	0.27	0.38	0.49	0.80	0.94	0.53	5822.08
2	Early	0.039	0.089	0.101	1.20	1.53	1.70	1.28	3.68	3.69	0.88	0.85	0.26	5010.39
	Late	0.019	0.126	0.246	1.87	2.65	2.06	1.30	4.69	8.09	0.83	0.82	0.29	3727.13
	Early	0.	0.	0.	1.71	2.25	2.22	0.46	0.52	0.51	0.90	0.96	0.56	5197.12
	Late	0.	0.	0.	2.86	3.27	2.44	0.22	0.47	0.54	0.79	0.79	0.31	3897.96

Figure Captions

Figure 1. Probability of responding with category A for each of the training items across training blocks for early, mid, and late conditions in Experiment 1. The legend identifies each item's reinforcement probabilities, $P(A|j)$, before and after the shift.

Figure 2. Observed slope through response probabilities across all 4 training items in Experiment 1 (solid lines) and objective slopes (dotted lines). Error bars indicate 95% confidence intervals. The three panels show the early (top), mid (middle), and late (bottom) condition, respectively.

Figure 3. Decisional-recency analysis for all conditions in Experiment 1. The probability of shifting the response from one category to the other on trial n is shown as a function of whether the response on trial $n-1$ was an error or correct. The top panel is for two identical stimuli in succession and the bottom panel is for two maximally dissimilar stimuli following each other. The smooth lines are based on locally-weighted regression (*lowess*). See text for details of alignment of conditions.

Figure 4. Observed slope through response probabilities across all 4 training items in Experiment 2 (solid lines) and objective slopes (dotted lines). Error bars indicate 95% confidence intervals. The two panels show the early (top) and late (bottom) condition, respectively.

Figure 5. Decisional-recency analysis for all conditions in Experiment 2. The probability of shifting the response from one category to the other on trial n is shown as a function of whether the response on trial $n-1$ was an error or correct. The top panel is for two identical stimuli in succession and the bottom panel is for two maximally dissimilar stimuli following each other. The smooth lines are based on locally-weighted regression (*lowess*). See text for details of alignment of conditions.

Figure A1. Observed slopes through response probabilities across all 4 training items in Experiment 1 (solid lines and error bars), slopes predicted by GCM (solid lines and open circles), and objective slopes (dotted lines). Error bars indicate 95% confidence intervals. The three panels show the early (top), mid (middle), and late (bottom) condition, respectively.

Figure A2. Observed slopes through response probabilities across all 4 training items in Experiment 2 (solid lines and error bars), slopes predicted by GCM (solid lines and open circles), and objective slopes (dotted lines). Error bars indicate 95% confidence intervals. The two panels show the early (top) and late (bottom) condition, respectively.

Figure A3. Observed slopes through response probabilities across all 4 training items in Experiment 1 (solid lines and error bars), slopes predicted by the MAC model (solid lines and open circles), and objective slopes (dotted lines). Error bars indicate 95% confidence intervals. The three panels show the early (top), mid (middle) and late (bottom) condition, respectively.

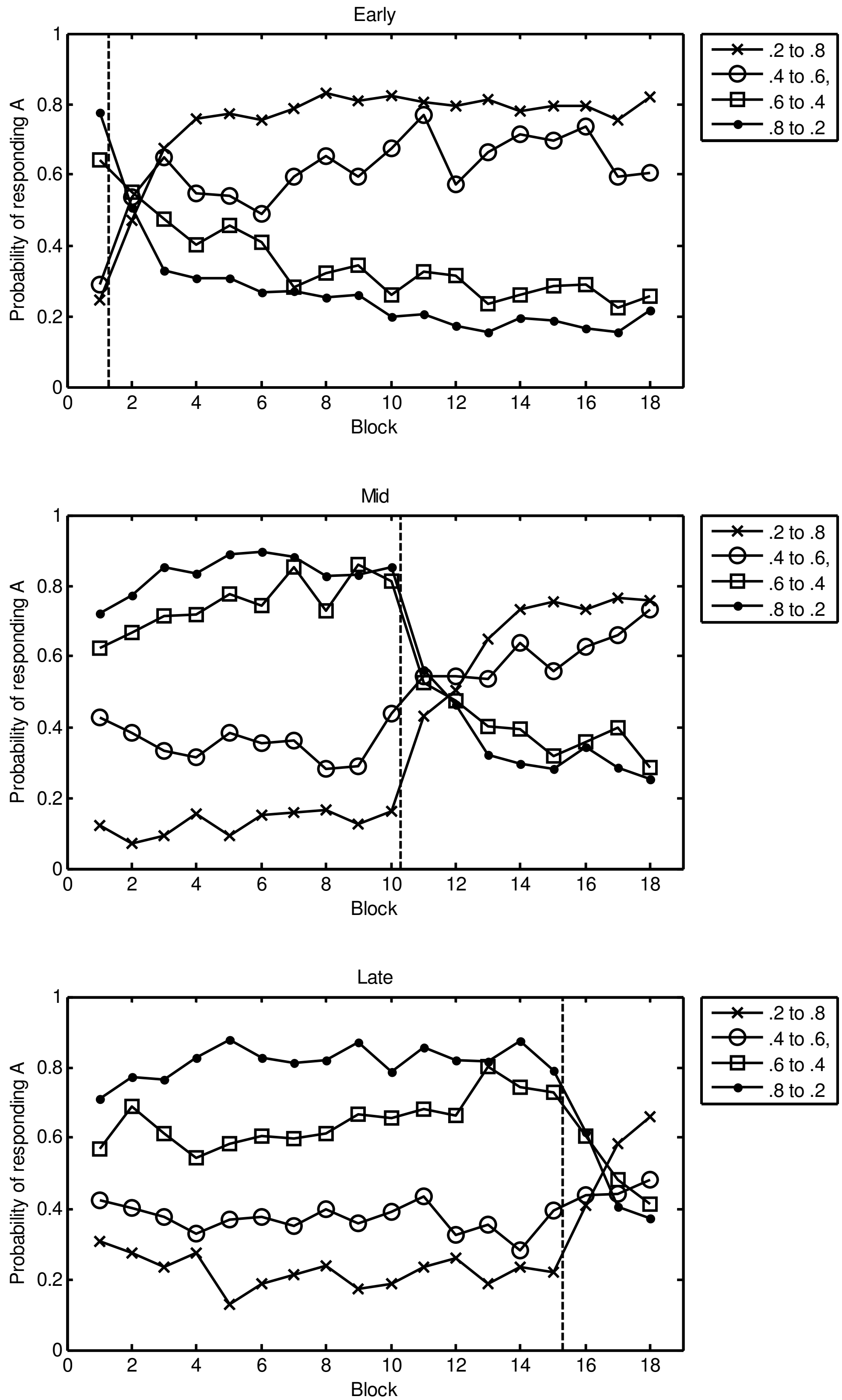
Figure A4. Observed slopes through response probabilities across all 4 training items in Experiment 2 (solid lines and error bars), slopes predicted by the MAC model (solid lines and open circles), and objective slopes (dotted lines). Error bars indicate 95% confidence intervals. The two panels show the early (top) and late (bottom) condition, respectively.

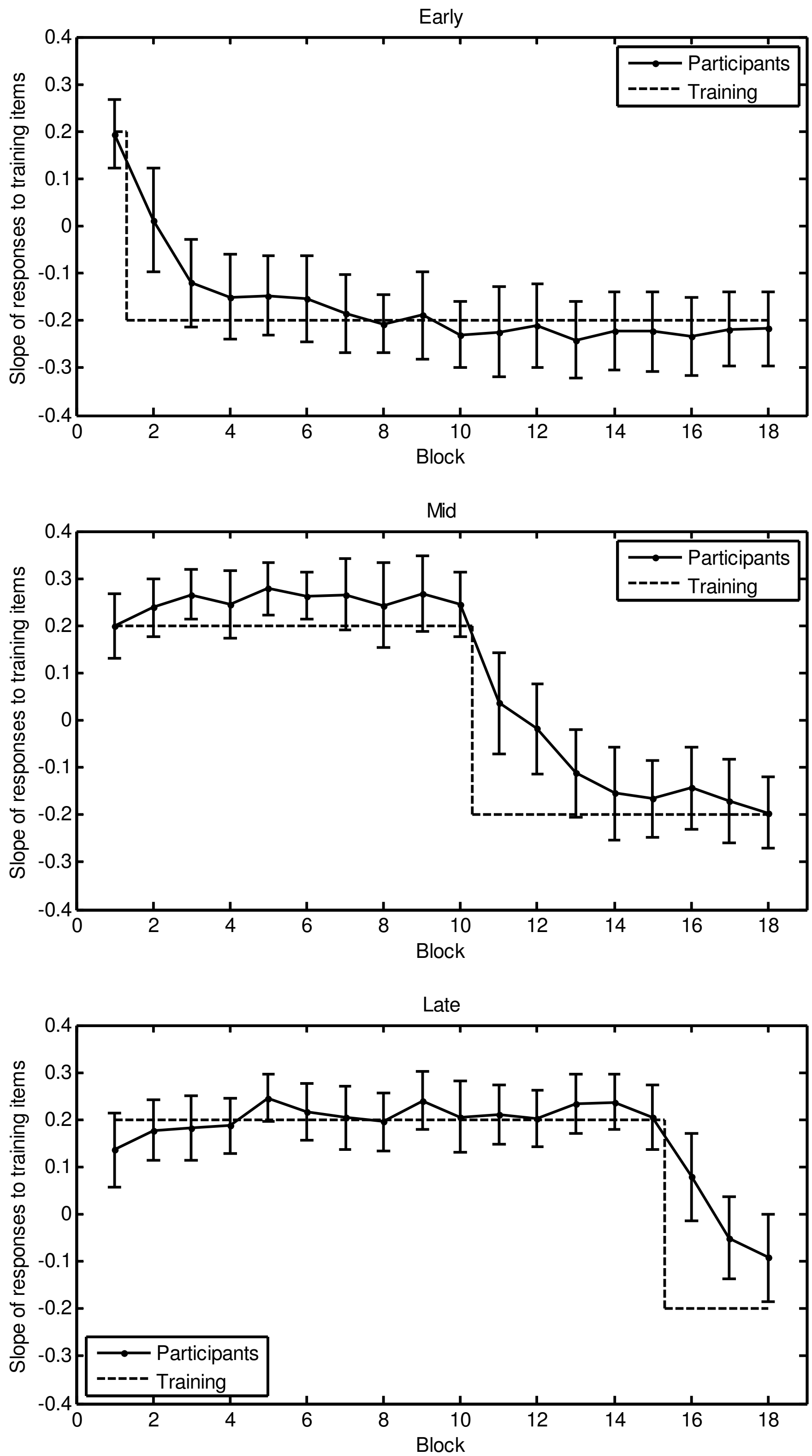
Figure A5. Observed slopes through response probabilities across all 4 training items in Experiment 1 (solid lines and error bars), slopes predicted by RASHNL (solid lines and open circles), and objective slopes (dotted lines). RASHNL's predictions were obtained with the annealing parameter, ρ, being freely estimated. Error bars indicate 95% confidence intervals. The three panels show the early (top), mid (middle), and late (bottom) condition, respectively.

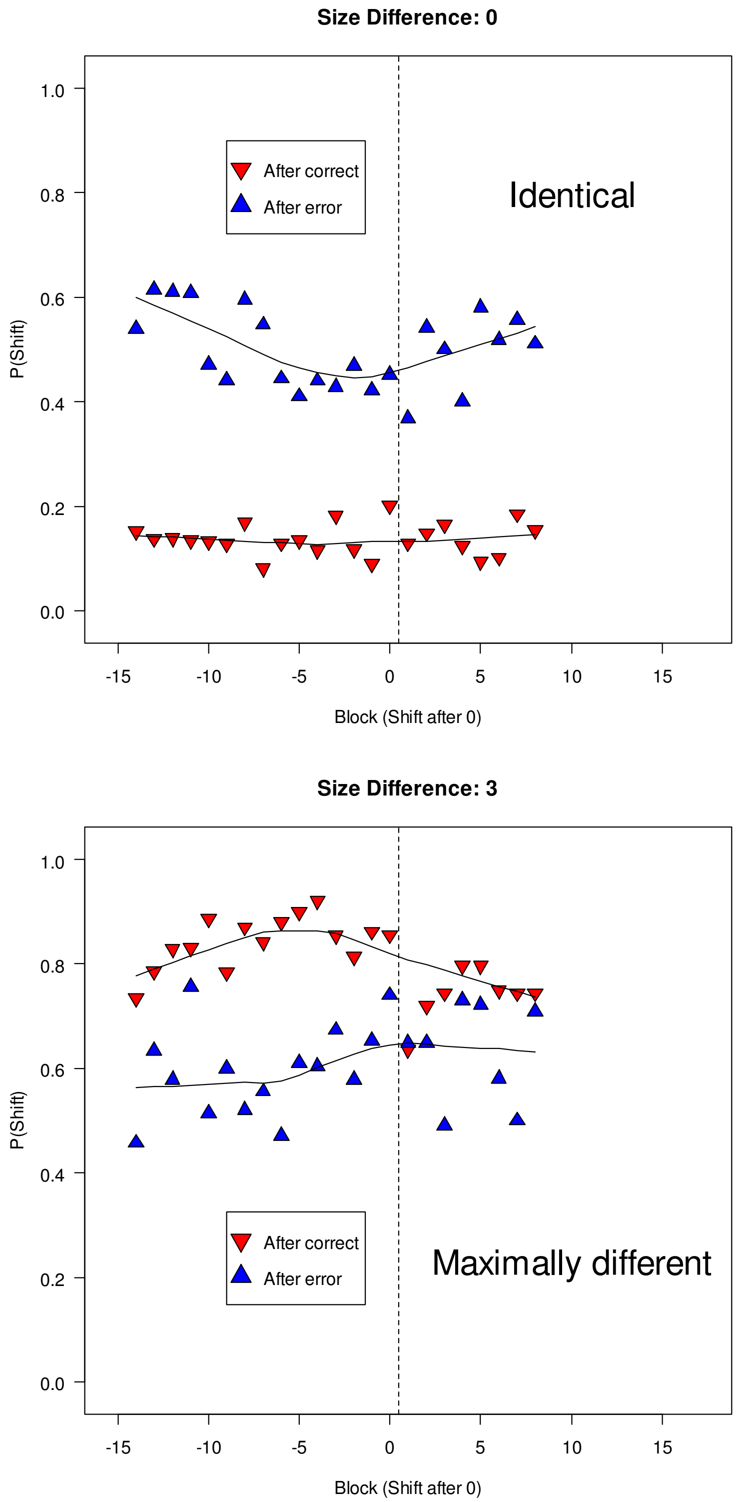
Figure A6. Observed slopes through response probabilities across all 4 training items in Experiment 2 (solid lines and error bars), slopes predicted by RASHNL (solid lines and open circles), and objective slopes (dotted lines). RASHNL's predictions were obtained with the annealing parameter, ρ, being freely estimated. Error bars indicate 95% confidence intervals. The two panels show the early (top) and late (bottom) condition, respectively.

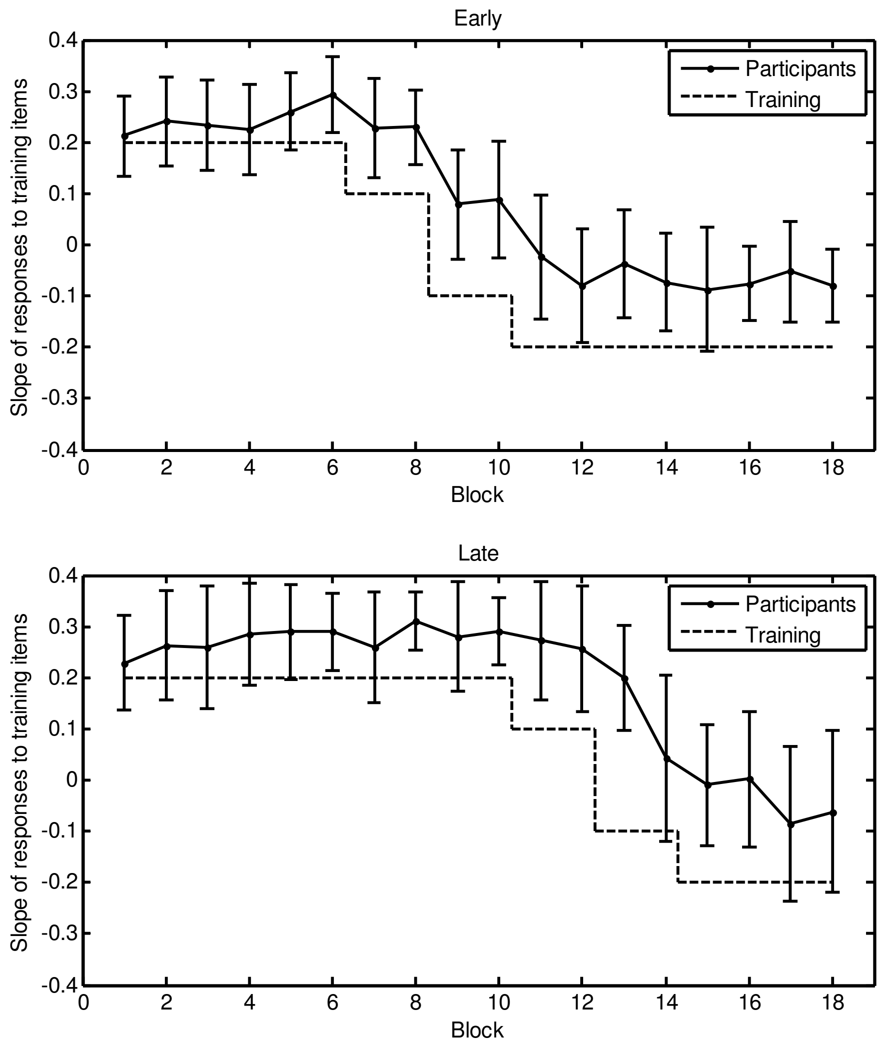
Figure A7. Observed slopes through response probabilities across all 4 training items in Experiment 1 (solid lines and error bars), slopes predicted by rATRIUM (solid lines and open circles), and objective slopes (dotted lines). rATRIUM's predictions were obtained with the annealing parameter, ρ, being freely estimated. Error bars indicate 95% confidence intervals. The three panels show the early (top), mid (middle), and late (bottom) condition, respectively.

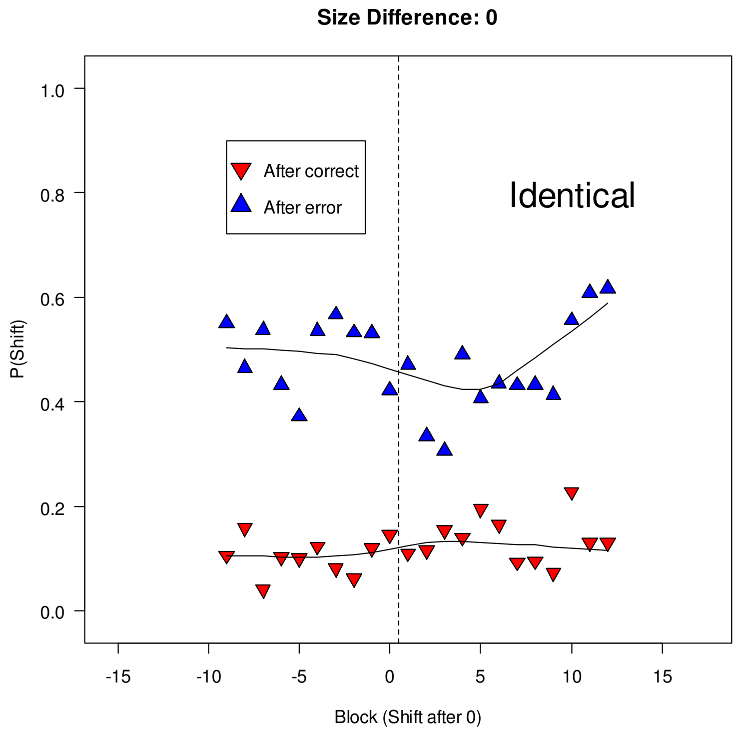
Figure A8. Observed slopes through response probabilities across all 4 training items in Experiment 2 (solid lines and error bars), slopes predicted by rATRIUM (solid lines and open circles), and objective slopes (dotted lines). rATRIUM's predictions were obtained with the annealing parameter, ρ, being freely estimated. Error bars indicate 95% confidence intervals. The two panels show the early (top) and late (bottom) condition, respectively.

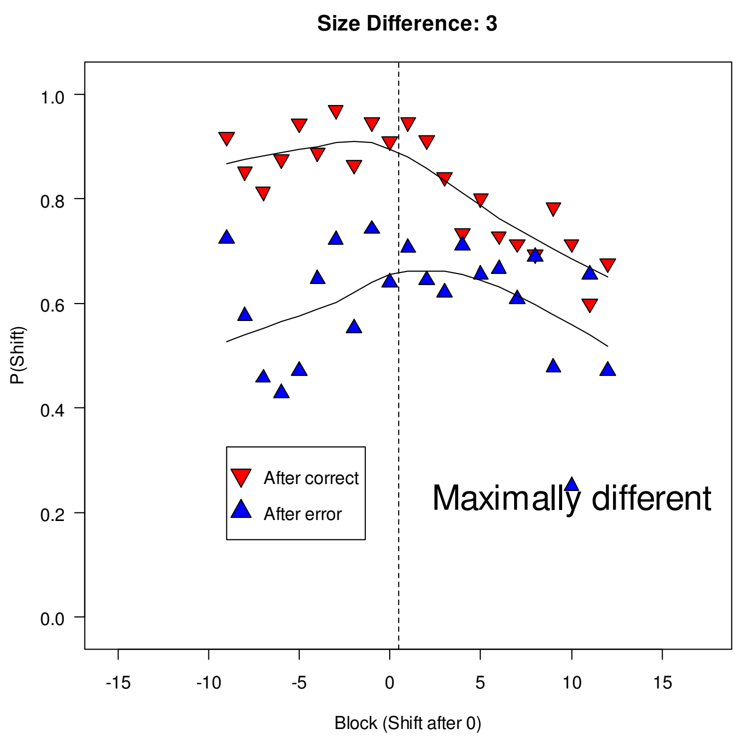

Early
Mid
Late
Probability of responding A
Block
.2 to .8
.4 to .6,
.6 to .4
.8 to .2

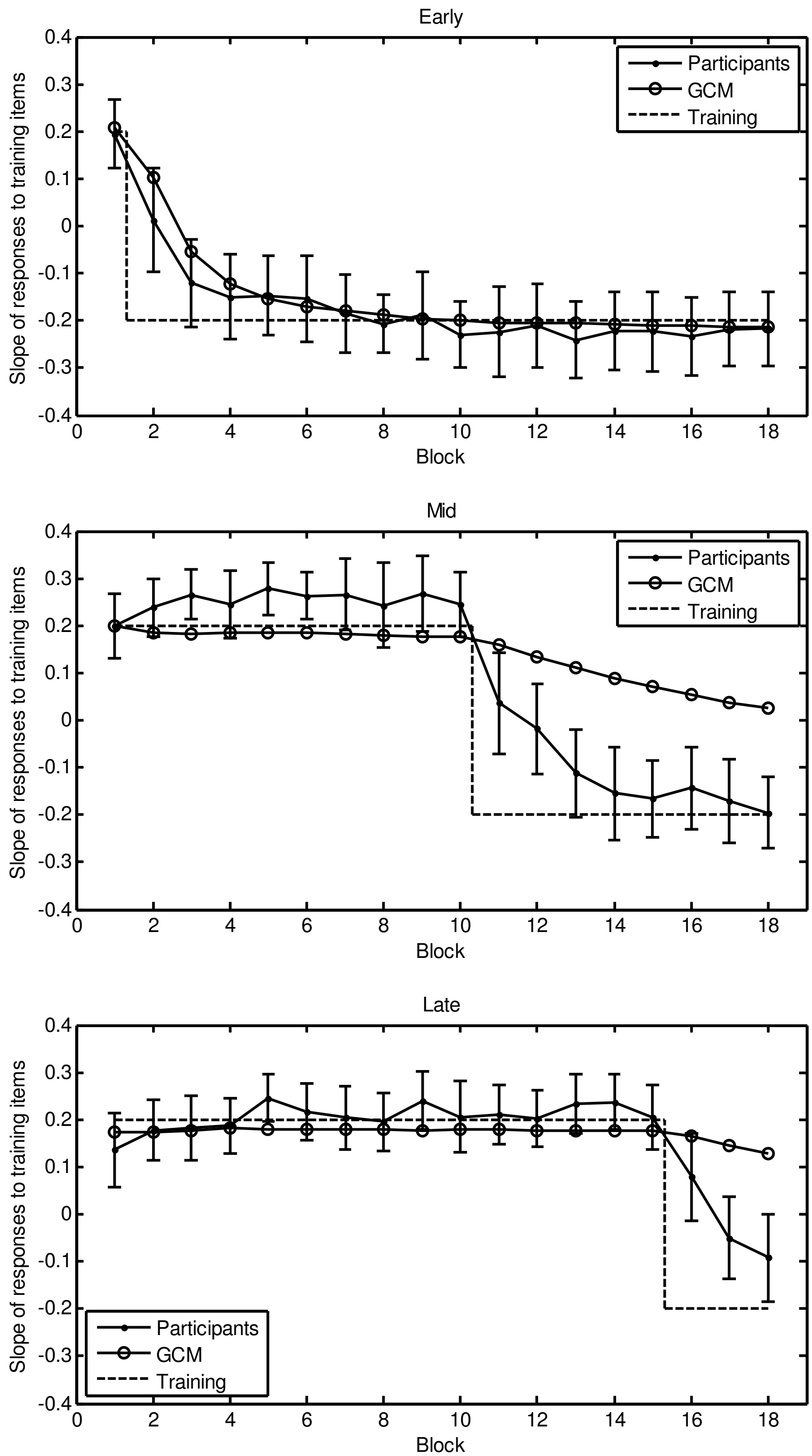

Early
Participants
Training
Slope of responses to training items
Block
Mid
Participants
Training
Slope of responses to training items
Block
Late
Participants
Training
Slope of responses to training items
Block

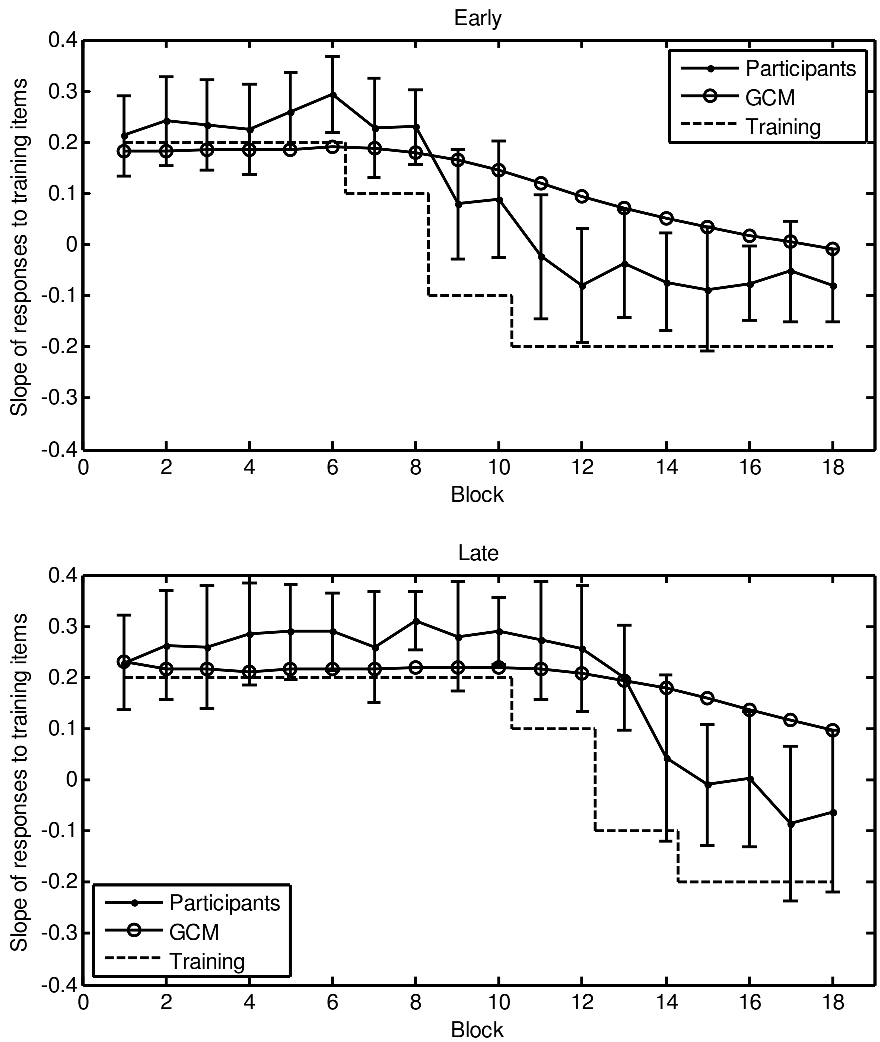

Size Difference: 0
After correct
After error
Identical
P(Shift)
Block (Shift after 0)
Size Difference: 3
After correct
After error
Maximally different
P(Shift)
Block (Shift after 0)

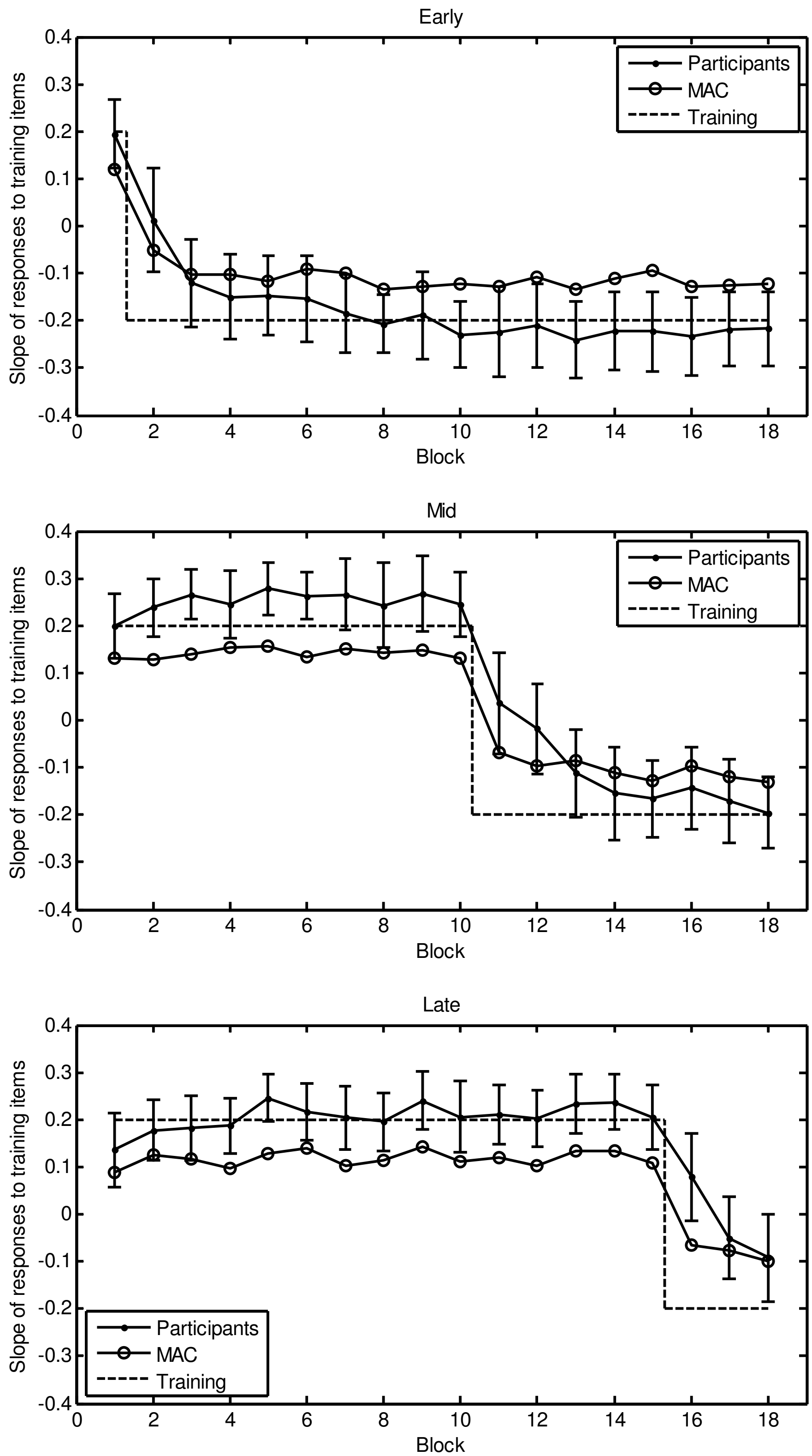

Early
Participants
Training
Slope of responses to training items
Block
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
2
4
6
8
10
12
14
16
18
Late
Participants
Training
Slope of responses to training items
Block

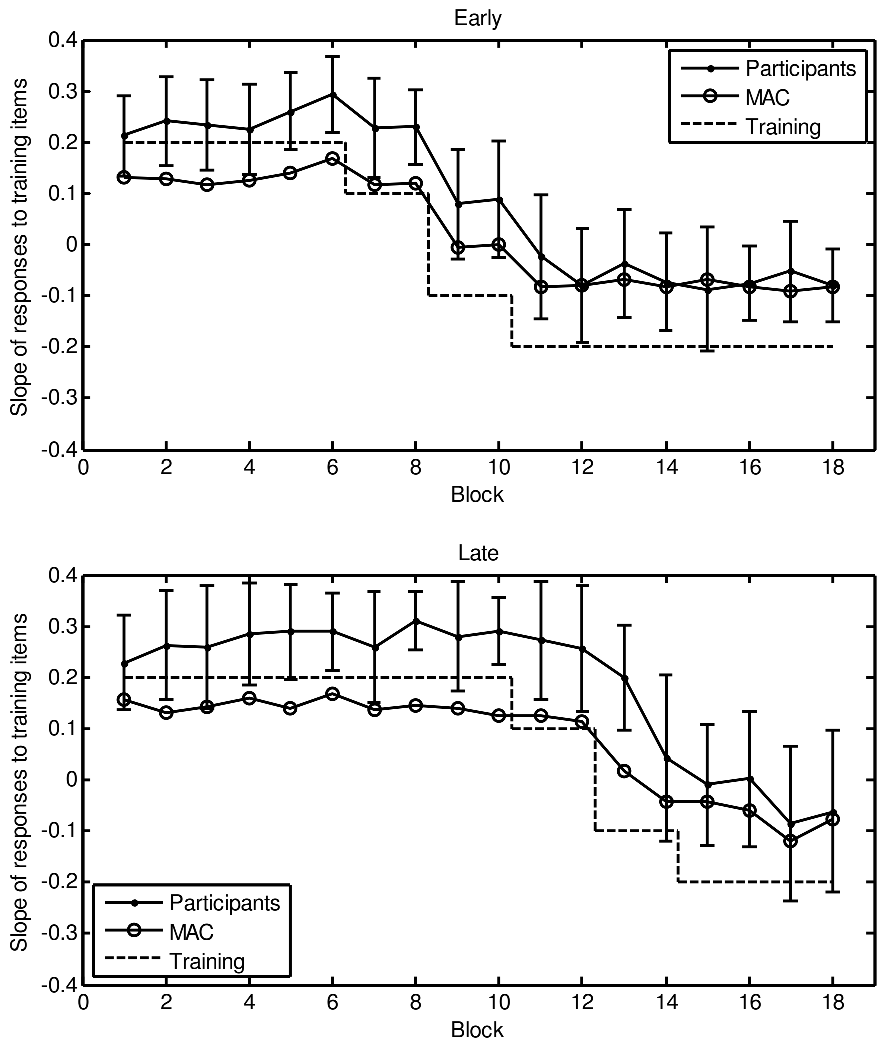

Size Difference: 0
After correct
After error
Identical
P(Shift)
Block (Shift after 0)
1.0
0.8
0.6
0.4
0.2
0.0
-15
-10
-5
0
5
10
15

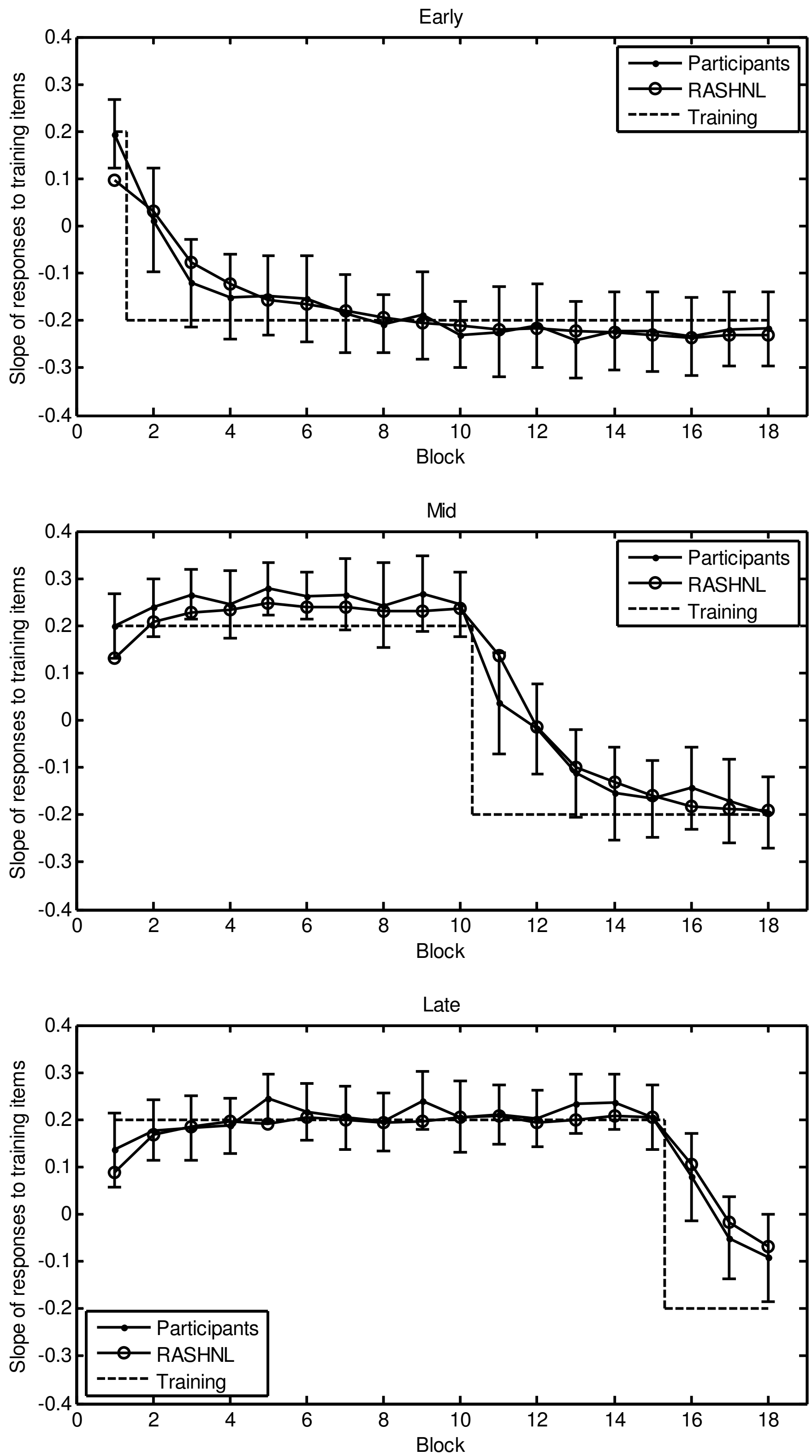

Size Difference: 3
After correct
After error
Maximally different
P(Shift)
Block (Shift after 0)
1.0
0.8
0.6
0.4
0.2
0.0
-15
-10
-5
0
5
10
15

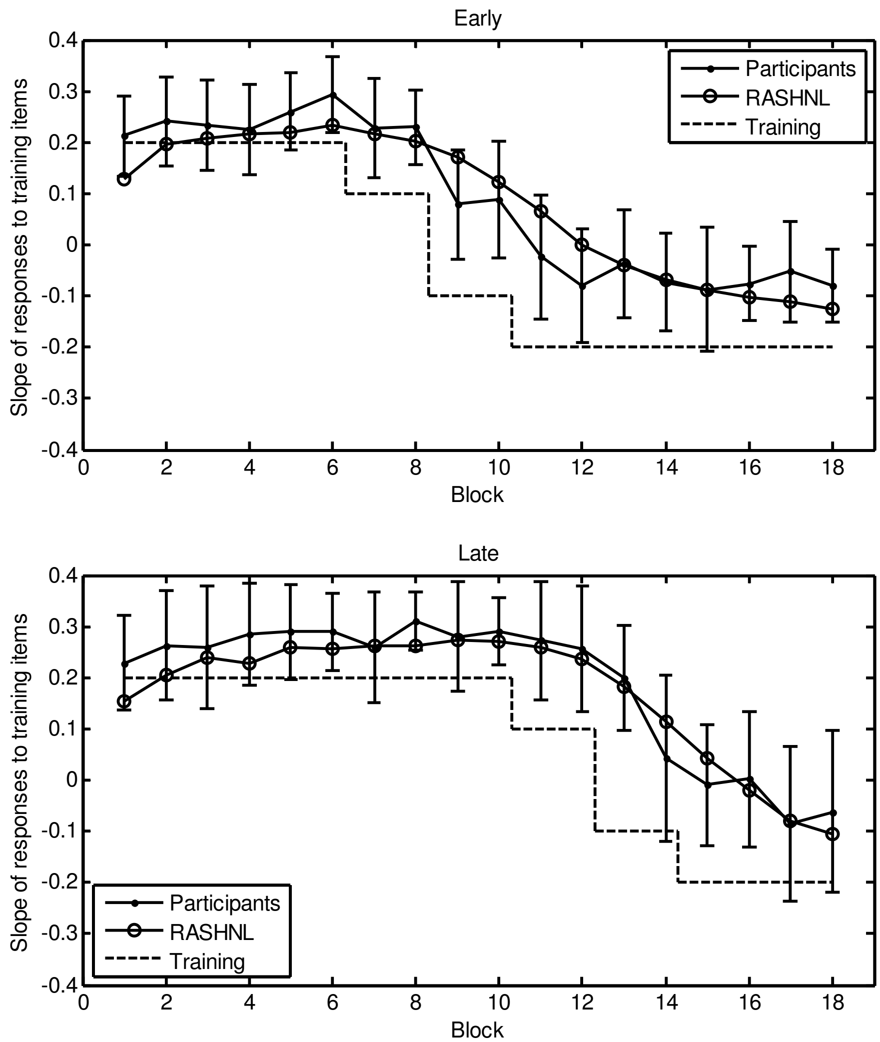

Early
Participants
GCM
Training
Slope of responses to training items
Block
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
2
4
6
8
10
12
14
16
18
Mid
Participants
GCM
Training
Slope of responses to training items
Block
Late
Participants
GCM
Training
Slope of responses to training items
Block

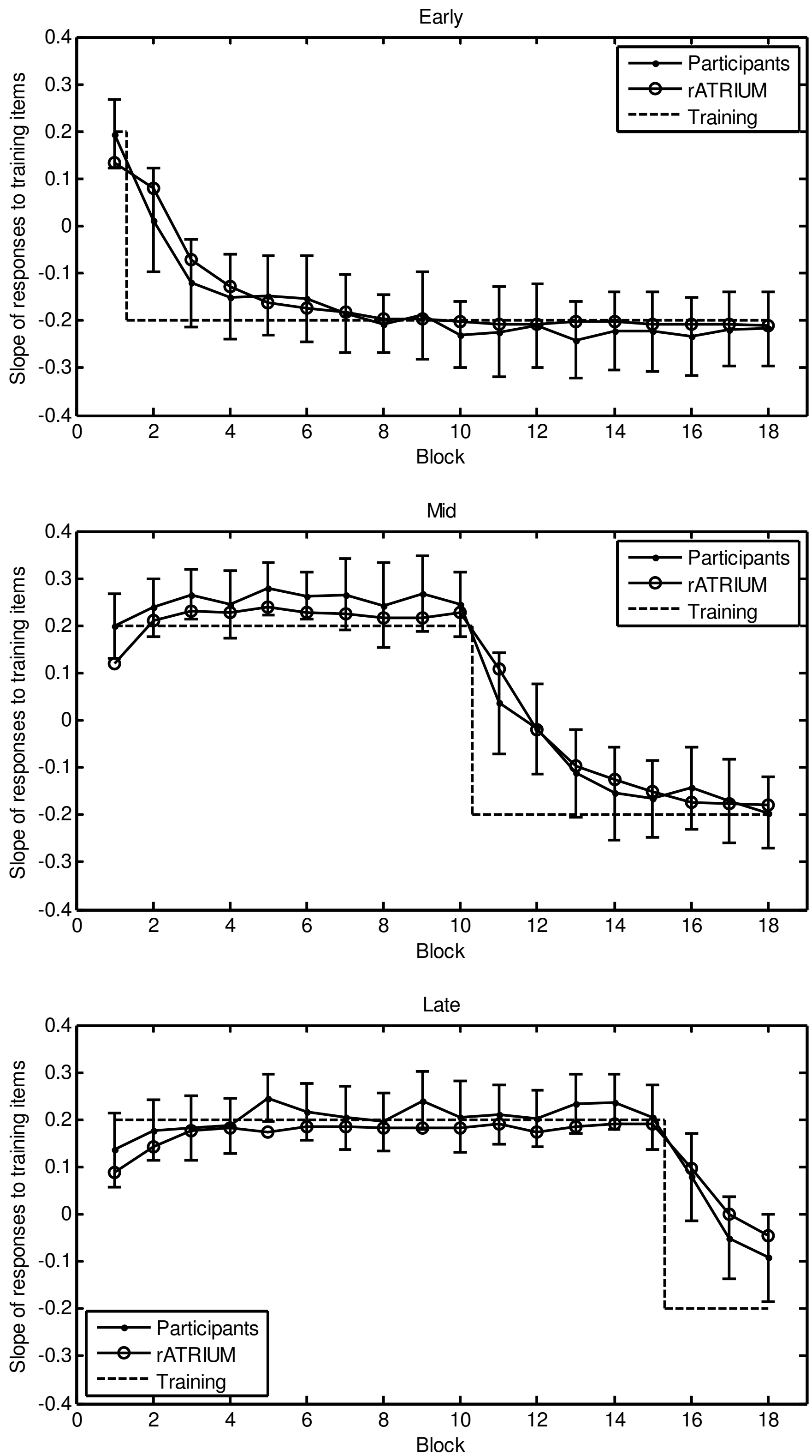

Early
Participants
GCM
Training
Slope of responses to training items
Block
Late
Participants
GCM
Training
Slope of responses to training items
Block

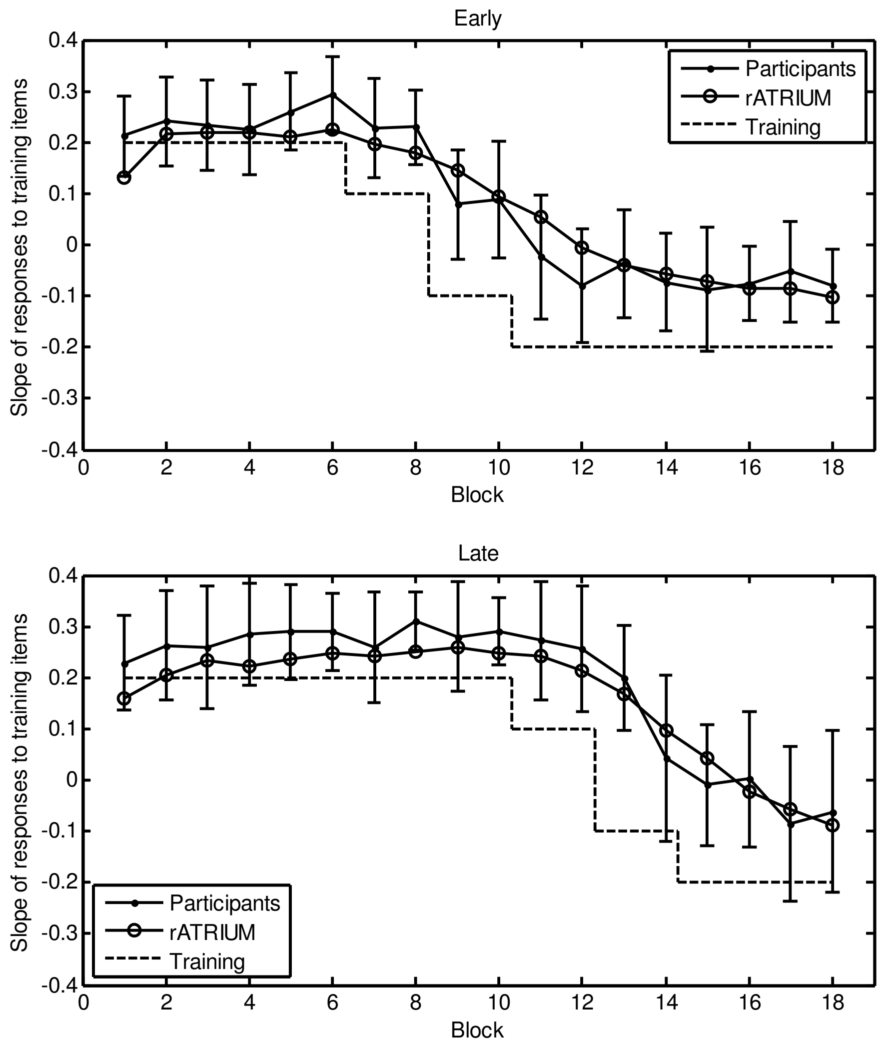

Early
Participants
MAC
Training
Slope of responses to training items
Block
Mid
Participants
MAC
Training
Slope of responses to training items
Block
Late
Participants
MAC
Training
Slope of responses to training items
Block


Early
Late
Slope of responses to training items
Block
Participants
MAC
Training
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
2
4
6
8
10
12
14
16
18


Early
Participants
RASHNL
Training
Slope of responses to training items
Block
Mid
Participants
RASHNL
Training
Slope of responses to training items
Block
Late
Participants
RASHNL
Training
Slope of responses to training items
Block


Early
Participants
RASHNL
Training
Slope of responses to training items
Block
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
2
4
6
8
10
12
14
16
18
Late
Participants
RASHNL
Training
Slope of responses to training items
Block


Early
Participants
rATRIUM
Training
Slope of responses to training items
Block
Mid
Participants
rATRIUM
Training
Slope of responses to training items
Block
Late
Participants
rATRIUM
Training
Slope of responses to training items
Block


Early
Participants
rATRIUM
Training
Slope of responses to training items
Block
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
2
4
6
8
10
12
14
16
18
Late
Participants
rATRIUM
Training
Slope of responses to training items
Block